Precautionary Mismatch*

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Abstract

How does wealth affect the extent to which the "right" workers are allocated to the "right" jobs? We study this question using a model with worker and firm heterogeneity, search frictions and incomplete markets. In the model, workers and firms jointly face a trade-off between the speed of match formation and the productivity of a match. As production-maximizing matches are hard to form due to search frictions, workers and firms agree on a range of mutually-acceptable matches. For workers having little wealth while searching for jobs, this trade-off is weighed in favor of speed due to precautionary motive, leading to weaker sorting and thus a higher degree of skill mismatch. We call this phenomenon "precautionary mismatch". We show that the model's predictions of the relationships between wealth, search behavior and labor market outcomes are consistent with empirical evidence from NLSY79 and O*NET. To shed light on the role of wealth in affecting labor market allocation and efficiency, we conduct a counterfactual exercise using a financial shock that erases 50% of wealth held by workers. We find that by exacerbating precautionary mismatch, the shock leads to a substantial decrease in productivity, especially for high-skilled workers.

Key words: Incomplete Markets, Search and Matching, Mismatch

JEL Codes: J64, E21, D31

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1 Introduction

It has been well recognized that factor misallocation is a key determinant of economic productivity. Since Hsieh and Klenow (2009), a large and growing literature has emphasized the role of capital misallocation across firms, which is usually conceptualized as the dispersion in marginal products across firms, and its resulting negative effects on aggregate productivity. This literature largely focuses on capital allocation, presumably because capital is more homogeneous and hence there is a natural notion of marginal product. We believe it is equally interesting to study the potential contribution from labor misallocation, which is much harder to study due to the presence of wild heterogeneity embedded in workers as well as the heterogeneity in jobs' requirements.

This paper aims at bridging the gap and stresses the allocation of talents. There are a variety of reasons why labors can be misallocated, such as information frictions (Guvenen et al. (2020)), barriers to entry (Hsieh et al. (2019)), housing constraints (Hsieh and Moretti (2019)), search frictions (Gautier and Teulings (2015)), and so on. We focus on search frictions and importantly the role of wealth in shaping the patterns of labor allocation and thus productivity. Approaching this question requires a model in which workers possessing different talents are allocated to jobs (or firms, which we use interchangeably due to model features) of different types, and frictions exist to prevent perfect sorting and generate mismatch. Additionally, wealth should play a role in the decisions of workers and firms in order to have a meaningful interaction with labor (mis)allocation.

We therefore propose a framework with three key elements. First, both workers and firms are endowed with heterogeneous skill types, and there is a production technology that combines skills on both sides to produce final output. The specification of the production function determines the nature of sorting that occurs in equilibrium. Second, labor market is frictional and meetings are random, so that it takes time to form a "good" match between unemployed workers and vacant firms. This suggests that a trade-off exists between the speed of forming a successful match and the productivity of a match, and thus in equilibrium there exists a range of mutually-acceptable matches. Third, workers are risk-averse and are only able to save or borrow in a risk-free asset to insure against unemployment risk. Therefore, workers with little wealth have strong precautionary motive, which induces them to accept a wider range of jobs to speed up job search at the cost of potentially lower wages and match productivity. Meanwhile, as firms offer lower wages to low-wealth workers, it is more profitable for them to match with poorer workers for any given skill type. This means that firms are also willing to accept a wider range of workers with low wealth. As a result, workers' precautionary motive leads to a wider range of mutually-acceptable matches and hence a higher level of skill mismatch, which we refer to as "precautionary mismatch".

The model generates several testable relationships between wealth, job search and labor market outcomes. Our empirical evidence relies on a data set that links NLSY79, a survey-based panel that contains rich information about several cohorts born in the late 1970s, and O*NET, which describes the characteristics (such as skill and knowledge requirements) of different occupations. We use occupation as a proxy to identify job types. To characterize heterogeneous workers and jobs, we follow Lise and Postel-Vinay (2020) and estimate worker skills and job skill requirements using principal component analysis (PCA) on a variety of worker and occupation characteristics. Based on the observed sorting patterns, we then define a notion of skill mismatch as the distance between worker skills and the skill requirements of matched jobs. We show empirical evidence for several implications generated by the model. First, the extent of mismatch is negatively correlated with liquid wealth. In particular, low-wealth workers spend less time being unemployed but are likely to experience higher levels of skill mismatch. Second, workers with lower wealth receive lower wages (controlling for skills). We show suggestive evidence that part of the negative relationship comes through skill mismatch.

Since wages, employment and wealth distribution are all determined in equilibrium, our model also features an endogenous joint distribution of wealth, wages and employment status, which is mostly absent or degenerate in existing models. Therefore in addition to the study of wealth and labor productivity, this model can also potentially be used as a tool to explore the interactions between wage and wealth inequality after careful calibration. However, at the current stage this is beyond our scope of study.

We highlight the importance of wealth in the determination of aggregate labor productivity through a counterfactual exercise, where we hit the model-generated stationary equilibrium with a wealth shock that erases 50% of wealth from all workers. The wealth shock leads to an increase in precautionary mismatch motive for all households, thereby exacerbating the amount of skill mismatch in the economy. We show that productivity decreases for all types of workers, but especially for high-skilled workers as they tend to suffer the most wealth decline and it is more costly for them to be mismatched.

Since model calibration is still under construction, we refrain from providing any quantitative results, but we would like to point out that a fully calibrate version of our model will be well-equipped to answer many fascinating questions. For example, by how much will our economy be more productive under a more equal wealth distribution? We can also answer policy-oriented questions such as how the current insurance policies such as unemployment insurance affect labor productivity.

Related Literature

Theoretically, our paper extends the linear utility assumption in a standard job search framework initiated from McCall (1970) by incorporating risk averse agents in an incomplete-markets model. The key insights arise from the standard exogenous income process being replaced by job search behavior that endogenizes uninsurable income risk. Using a Diamond-Mortensen-Pissarides framework with risk aversion, Krusell, Mukoyama and Şahin (2010) is the first to study an incomplete-markets model with labor-market frictions, which is used to evaluate a tax-financed unemployment insurance scheme. Lise (2013) introduces on-the-job search but focuses on a partial equilibrium, and generates an important asymmetry of saving behavior between the incremental wage increases generated by on-the-job search (climbing the wage ladder) and the drop in income associated with job loss (falling off the ladder). Recent updates including Eeckhout and Sepahsalari (2018), Chaumont and Shi (2018), Herkenhoff, Phillips and Cohen-Cole (2017) and Krusell, Luo and Rios-Rull (2019), instead study a directed search equilibrium model with risk-averse workers, where the key trade-off is the speed of finding a job versus the wage for workers (and similarly, the speed of filling a vacancy versus profits for firms). Griffy (2018) further introduces human capital accumulation to study the life-cycle inequality in earnings and wealth. Our framework organically nests three strands of the macro and labor literature: an assignment model by Becker (1973), a Diamond-Mortensen-Pissarides search and matching model, and an incomplete-markets model in the spirit of Bewley (1977)-Huggett (1993)-Aiyagari (1994). We show that the framework features two limiting economies: without two-sided heterogeneity, our model is the same as Krusell, Mukoyama and Şahin (2010) in which wealth and wages correspond one-to-one; without risk aversion, our model becomes Shimer and Smith (2000) in which workers possess different skills but not wealth. In this regard, we also contribute to the literature featuring search and matching with two-sided heterogeneity such as Dolado, Jansen and Jimeno (2009) and Bagger and Lentz (2019).

Our computation strategy is inspired by Achdou et al. (2020), which uses a continuous-time approach to cast rather complex optimization problems and equilibrium conditions in incomplete-markets models into two coupled systems of partial differential equations that are easier to compute. In our model, the continuous-time approach also enables us to write down intuitive expressions for equilibrium wages and consumption-saving policies, which further reduces the difficulty of computation[1] and facilitates understanding of our model's properties.

Empirically, our paper is related to a large literature documenting relations between asset holdings and job search behavior (see, for example, Card, Chetty and Weber (2007), Rendon

[1] Krusell, Mukoyama and Şahin (2010) point out that computation of an equilibrium where assets enter Nash bargaining problem is difficulty in discrete time.

(2007), Lentz (2009), Chetty (2008), Herkenhoff, Phillips and Cohen-Cole (2017), among many others). These papers show overwhelmingly that increasing the ability to smooth consumption, either through unemployment insurance, wealth or access to credit, leads to longer unemployment duration and higher accepted wages. These findings provide us with an important guidance to think about the implications of the observed search behavior in the context of labor market sorting. A natural prediction from a longer unemployment duration is that the match quality of unemployed workers with new jobs also increases. To our knowledge, we are among the first papers to document the joint effect of worker assets and skills on allocations to jobs following an unemployment spell. Our approach to measure worker and job heterogeneity follows recent papers including Lise and Postel-Vinay (2020) and Guvenen et al. (2020), which also use observable worker and job characteristics from NLSY79 and O*NET to estimate skills mismatch and effects on wages. We extend their approach to include wealth heterogeneity and show that skills mismatch is likely to be influenced by precautionary motive.

Additionally, our methodology is inspired by a recent literature that studies multidimensional skills mismatch. Lindenlaub and Postel-Vinay (2020) characterizes sorting with random search when both workers and jobs have multi-dimensional heterogeneity. Their key theoretical insight is that multi-dimensional heterogeneity is in itself a source of sorting. They also argue that multi-dimensional sorting is empirically relevant in the sense that a single-index representation misses substantial features in the data. Lise and Postel-Vinay (2020) studies dynamic sorting by incorporating human capital accumulation via learning by doing. An interesting finding is that the half-life of skill accumulation varies quite a lot across different types of skills. According to their estimates, the half-year is 7.5 for cognitive skills, 1.7 for manual skills, and 55.8 for interpersonal skills. The message is that it is super hard to accumulate one's interpersonal skills. Closely related is Guvenen et al. (2020) who also examines multidimensional skill mismatch using similar empirical measures, but provides a different theoretical angle for the source of mismatch. In Lise and Postel-Vinay (2020), the source of mismatch is the (random) search frictions, while in Guvenen et al. (2020) it is the misperception about one's own abilities. Baley, Figueiredo and Ulbricht (2019) studies the business cyclic properties of mismatch.

Lastly, we contribute to the macro-development literature on misallocation, which is pioneered by Restuccia and Rogerson (2008) and Hsieh and Klenow (2009). The idea is so appealing – if we reallocate some production from a firm with a lower marginal product to a firm with a higher marginal product, we could achieve higher aggregate output even without accumulating any inputs. This literature, by its nature, pays more attention on the firm side and typically abstracts away from labor heterogeneity. Our paper digs deeper into the misallocation arising from the allocation of heterogeneous workers to heterogeneous firms.

The rest of the paper is organized as follows. In Section 2, we describe the model and the algorithm to solve it. In Section 3, we discuss several key theoretical results regarding the connections between wealth, job search behavior and labor market outcomes. In Section 4, we describe the data sets we use for empirical analysis and the methods to estimate worker and firm types. In Section 5 we present empirical evidence on the relationship between liquid wealth, skill mismatch and wages. In Section 6 we show model calibrations and counterfactual exercises. Section 7 concludes.

2 Model

2.1 Environment

Time is continuous and there is no aggregate uncertainty. We assume that there is a unit measure of workers that are infinitely-lived.

Preference. Workers maximize expected present value according to a common discount rate ρ, and jobs maximize the present value of expected profits discounted at rate r, equal to the risk-free interst rate of the economy. Workers are risk averse with flow utility $\mathfrak{u}(c)$ and firms are risk neutral. The utility function $\mathfrak{u}(\cdot)$ exhibits common properties $\mathfrak{u}' > 0, \mathfrak{u}'' < 0$.

Production. Workers and jobs are heterogeneous. Workers are characterized by skill type $x \in \mathbb{X}$ and jobs by skill requirement type $y \in \mathbb{Y}$. We normalize $\mathbb{X}$ and $\mathbb{Y}$ to unit intervals. The production function of a matched pair is denoted $f(x,y) : \mathbb{X} \times \mathbb{Y} \to \mathbb{R}_+$. We impose technical assumptions on f to guarantee existence. Unemployed workers produce b (e.g., leisure, unemployment benefits, and home production).

Search and Matching. Labor markets are frictional. Search and matching is random via a *meeting* function $M(u,v)$ that is constant returns to scale (CRS), where u denotes unemployment and v vacancies. We denote by $\theta = v/u$ the labor market tightness. Due to CRS, the meeting rate for an unemployed worker can be written as $p(\theta) := M(u,v)/u = M(1,\theta)$. Similarly, the meeting rate for a vacancy can be written as $q(\theta) := M(u,v)/v = M(\theta^{-1},1)$. Note that $q(\theta) = p(\theta)/\theta$. The difference between *meetings* and successful *matches* is worth noting. Once a worker and a job meet, they can decide whether to start production or not. Some meetings may not end up with a successful match if the agents prefer to continue searching. Jobs are destroyed exogenously with a Poisson rate σ. In the benchmark model, there is no on-the-job search. Wage is determined by Nash bargaining with worker bargaining power denoted η.

Incomplete Market. There is not a complete set of Arrow securities. Instead, there is only one asset that agents can save at a risk-free rate r to smooth consumption against fluctuations in labor income. Workers face a borrowing constraint $\underline{a}$.

2.2 Characterization

2.2.1 Distribution

Before characterizing the value functions, it proves useful to define several relevant measures. The population distributions over worker types and job types are given by $d_w(x)$ and $d_j(y)$, respectively. For the convenience of notations, we refer to matches as m, employed workers e, unemployed workers u, producing jobs p, and vacant jobs v, all using the first letter the words. For example, the density function of producing matches is denoted $d_m(a,x,y) : \mathbb{R} \times \mathbb{X} \times \mathbb{Y} \to \mathbb{R}_+$. We could define other densities in a similar fashion, with density of employed workers $d_e(a,x) = \int d_m(a,x,y)\,\mathrm{d}y$, density of unemployed workers $d_u(a,x)$, density of producing jobs $d_p(y) = \iint d_m(a,x,y)\,\mathrm{d}a\mathrm{d}x$, and density of vacant jobs $d_v(y) = d_j(y) - d_p(y)$. Notice that the aggregate unemployment and vacancy are given by $u = \iint d_u(a,x)\,\mathrm{d}a\mathrm{d}x$ and $v = \int d_v(y)\,\mathrm{d}y$, respectively. These add-up properties are summarized in Table 1.

Table 1: Distribution Add-up Properties

Description	Add-up Property
Workers	$d_w(x) = \int d_u(a,x)\,da + \iint d_m(a,x,y)\,\mathrm{d}y da$
Total unemployment	$u = \iint d_u(a,x)\,\mathrm{d}x\mathrm{d}a$
Firms	$d_j(y) = d_v(y) + \iint d_m(a,x,y)\,\mathrm{d}x\mathrm{d}a$
Total vacancies	$v = \int d_v(y)\,\mathrm{d}y$

Notes: The table summarizes the aggregation properties relating densities $d_u(a,x)$, $d_v(y)$, u, v and the match density $d_m(a,x,y)$.

2.2.2 Hamilton-Jacobi-Bellman Equations

Worker Values

Let $U(a,x)$ denote the value of an unemployed worker of type x with with wealth a, and $W(a,x,y)$ the value of an employed worker of type x with asset a working at a firm of type

y. The HJB equation for the value of being employed is:

$$\rho W(a,x,y) = \max_c \mathrm{u}(c) + \sigma\left[U(a,x) - W(a,x,y)\right] + \dot{a}W_a(a,x,y) \tag{1}$$
$$\text{s.t. } \dot{a} = ra + \omega(a,x,y) - c$$
$$a \geq \underline{a}$$

where $[\bullet]^+ := \max\{\bullet, 0\}$. An employed receives flow interest ra and wage $\omega(a,x,y)$, and makes a consumption-saving decision, which gives a flow utility of $\mathrm{u}(c)$. With Poisson rate σ, the worker loses the job and becomes unemployed. The optimal consumption-saving decision is characterized by the first order condition

$$\mathrm{u}'(c^e) = W_a(a,x,y). \tag{2}$$

The Hamilton-Jacobi-Bellman (HJB) equation for the value of being unemployed is:

$$\rho U(a,x) = \max_c \mathrm{u}(c) + p(\theta)\int \frac{d_v(y)}{v}\left[W(a,x,y) - U(a,x)\right]^+ \mathrm{d}y + \dot{a}U_a(a,x) \tag{3}$$
$$\text{s.t. } \dot{a} = ra + b - c$$
$$a \geq \underline{a}$$

An unemployed worker makes home production b as well as flow interest ra. The unemployed worker meets a vacant job at rate $p(\theta)$, which is randomly sampled from the distribution of all vacancies. The first order condition for the consumption-saving decision is given by

$$\mathrm{u}'(c^u) = U_a(a,x). \tag{4}$$

Firm Values

Let $V(y)$ denote the value of a vacant job of type y, and $J(a,x,y)$ the value of a producing job of type y, with an employee of type x who has asset a. The HJB equation for the producing job is

$$\rho J(a,x,y) = f(x,y) - \omega(a,x,y) + \sigma\left[V(y) - J(a,x,y)\right] + \dot{a}^e J_a(a,x,y), \tag{5}$$

where $\dot{a}^e := ra + \omega(a,x,y) - c^e(a,x,y)$ is the optimal saving policy of the employee. The firm retains the remaining output net of wage paid to the worker. With Poisson rate σ, the match is separated.

The value of a vacant job is

$$\rho V(y) = q(\theta) \iint \frac{d_u(a,x)}{u} [J(a,x,y) - V(y)]^+ \, dadx. \tag{6}$$

The vacancy meets an unemployed worker at rate $q(\theta)$ that is randomly drawn from the distribution of all unemployed workers.

The mass of jobs is determined by a free-entry condition. We assume that entrepreneurs need to pay a fixed entry cost of κ before the skill requirement type y is realized according to a cumulative density function $G(.)$.

$$\kappa = \int V(y) \, dG(y). \tag{7}$$

In equilibrium, the value of firms adjust so that the expected value of entry is zero.

2.2.3 Wage Determination

Wages are determined by Nash bargaining, denoted by $\omega(a,x,y)$. Appendix A.1 proves that the Nash solution can be characterized by

$$\eta \frac{J(a,x,y) - V(y)}{1 - J_a(a,x,y)} = (1-\eta) \frac{W(a,x,y) - U(a,x)}{W_a(a,x,y)}, \tag{8}$$

where $\eta \in (0,1)$ represents the bargaining power of the worker. In addition, we can derive an expression for wages (Appendix A.1, equation (15)) which can be easily computed once we obtain the value and policy functions of workers and firms.

To gain intuitions, it is useful to contrast our result with common Nash solutions in the case of linear utility. In an environment with linear utility, the match surplus is defined by the sum of worker's surplus and the job's surplus:

$$S(a,x,y) := W(a,x,y) - U(a,x) + J(a,x,y) - V(y).$$

Then Nash bargaining works in a way that the worker and the job are splitting the match surplus according to η. However, it does not make sense to directly add up the worker surplus and the firm surplus if they are not measured in the same units, as is in the case when we have curvature in the utility function. In particular, once we introduce curvature in the flow utility function, worker values are measured in present discounted util, while firm values are measured in present discounted numeraire. It turns out that W_a and $1 - J_a$ are the right adjustment terms so that we could add up the adjusted worker value and firm value. That is, consider the adjusted surplus

$$\hat{S}\left(a,x,y\right) := \frac{1}{W_a(a,x,y)}\left[W(a,x,y) - U\left(a,x\right)\right] + \frac{1}{1 - J_a(a,x,y)}\left[J\left(a,x,y\right) - V\left(y\right)\right]. \tag{9}$$

The worker and the firm are splitting the adjusted surplus according to the bargaining power η.

It is obvious that W_a properly measures the marginal value of a dollar to the worker. Now we illustrate the intuition why $1 - J_a$ is the right adjustment term for the firm. Think of the scenario of a marginal dollar transfer between the worker and the firm. If the worker transfers one additional dollar to the firm, there is a direct 1 dollar increase in firm's value and an indirect impact to the firm through asset decumulation of the worker, i.e., $-J_a$. Thus the total marginal value of additional dollar to the firm is properly captured by $1 - J_a$.

In the formal proof in Appendix A.1, we write down the full Nash problem by defining values of deviating wages with tilde notations, e.g., $\tilde{W}\left(w,a,x,y\right)$. We show that

$$\frac{-\tilde{J}_w(w,a,x,y)}{\tilde{W}_w(w,a,x,y)} = \frac{1 - J_a(a,x,y)}{W_a(a,x,y)},$$

which implies that the adjusted surplus could alternatively be written as

$$\hat{S}\left(a,x,y\right) := \frac{1}{\tilde{W}_w}\left[W(a,x,y) - U\left(a,x\right)\right] + \frac{1}{\left(-\tilde{J}_w\right)}\left[J\left(a,x,y\right) - V\left(y\right)\right].$$

This provides further intuition to the bargaining solution – the worker's surplus is adjusted by $\tilde{W}_w$ to the dollar value, and the firm's surplus is adjusted by $\left(-\tilde{J}_w\right)$ to the dollar value. Workers and firms split the adjusted surplus.

Finally, notice that as the curvature of the utility function goes to 0, i.e., as the utility function goes to linear, then $W_a = 1$ and $J_a = 0$. In this case, our adjusted surplus collapses to the standard definition of surplus.

2.2.4 Steady State

We consider a stationary equilibrium. The steady state could be characterized by two sets of Kolmogorov Forward (KF) equations. The first one characterizes the inflow-outflow balancing equation for employed workers $d_m\left(a,x,y\right)$, i.e.,

$$0 = -\frac{\partial}{\partial a}\left[\dot{a}^e\left(a,x,y\right)d_m\left(a,x,y\right)\right] - \sigma d_m\left(a,x,y\right) + d_u(a,x)p\left(\theta\right)\frac{d_v\left(y\right)}{v}\Phi\left(a,x,y\right), \tag{10}$$

for all a, x, y. The second one characterizes the inflow-outflow balancing equation for unemployed workers $d_u(a,x)$, i.e.,

$$0 = -\frac{\partial}{\partial a}\left[\dot{a}^u\left(a,x\right)d_u\left(a,x\right)\right] - \int p\left(\theta\right)\frac{d_v\left(y\right)}{v}\Phi\left(a,x,y\right)d_u(a,x)\mathrm{d}y + \sigma\int d_m\left(a,x,y\right)\mathrm{d}y, \qquad (11)$$

for all a, x. In addition, there is an add-up condition that density integrates to 1:

$$1 = \int_{\underline{a}}^{\infty} d_m\left(a,x,y\right)dadxdy + \int_{\underline{a}}^{\infty} d_u\left(a,x\right)dadx$$

as well as

$$d_x = \int_{\underline{a}}^{\infty} d_m\left(a,x,y\right)dady + \int_{\underline{a}}^{\infty} d_u\left(a,x\right)da$$

$$d_y = \int_{\underline{a}}^{\infty} d_m\left(a,x,y\right)dadx + d_v\left(y\right)$$

2.3 Equilibrium

2.3.1 Formal Equilibrium Definition

A stationary search equilibrium consists of a set of value functions $\{W\left(a,x,y\right), U\left(a,x\right), J\left(a,x,y\right), V\left(y\right)\}$ for employed workers, unemployed workers, producing jobs, and vacant jobs, respectively; a set of policy functions including consumption policy $\{c^e\left(a,x,y\right), c^u\left(a,x\right)\}$ and matching acceptance decision conditonal on meeting $\Phi\left(a,x,y\right)$; a wage policy $\omega\left(a,x,y\right)$; and an invariant distribution of employed workers $d_m\left(a,x,y\right)$ and unemployed workers $d_u\left(a,x\right)$, and market tightness θ such that:

1. The value functions and policy functions solve worker and firm's optimization problem (1, 3, 5, 6);
2. Wage setting and matching acceptance decision satisfy Nash bargaining (8);
3. The stationary distributions satisfy the Kolmogorov Forward equations (10 and 11);
4. Market tightness adjusts so that free entry condition in equation (7) gives zero economic profits to firms prior to entry.

2.3.2 Model Outputs

This model provides a joint characterization of employment, wages, and wealth distributions.

First, it characterizes standard labor market variables of interest. Since the baseline model assumes an exogeneous separation rate, it is thus the job losing rate in the economy $\pi_{eu} = \sigma$. The job finding rate (not job meeting) in the economy is

$$\pi_{ue} = p(\theta) \int \frac{d_v(y)}{v} \Phi(a, x, y) \, dy.$$

The steady state unemployment rate is given by

$$u = \frac{\sigma}{\sigma + \pi_{ue}},$$

which is known as the Beveridge curve.

Second, the model allows for a joint characterization of wage and wealth distributions. Specifically, the joint distribution of wealth and wage (among employed workers) is characterized by

$$h(a, w) = \frac{1}{e} \iint d_m(a, x, y) \mathbb{1} \{\omega(a, x, y) = w\} \, dx dy.$$

Third, the model allows us to study the determinants of aggregate output and productivity. The total output and measure of employed is

$$y = \iiint f(x, y) \, d_m(a, x, y) \, da dx dy,$$
$$e = \iiint d_m(a, x, y) \, da dx dy$$

and average output per employed (i.e. productivity) is $\bar{y} = y/e$. Since wealth distribution affects the allocation of worker types x to firm types y (as we will show below), our model allows us to study how wealth inequality affects labor market sorting and labor misallocation, characterized by output loss or productivity loss relative to a benchmark economy (e.g. without search friction or with complete markets).

2.4 Algorithm

Consider grids $\{a_1, a_2, \ldots, a_{N_a}\}$ for asset, $\{x_1, x_2, \ldots, x_{N_x}\}$ for skills,, $\left\{y_1, y_2, \ldots, y_{N_y}\right\}$ for skill requirements. Suppose they are equally spaced (probably assets are log-spaced) and $\Delta_a, \Delta_x, \Delta_y$ are the steps.

Since we only consider a stationary equilibrium, we normalize d_x, d_y to be uniform on $[0, 1]$.

1. Guess θ and $d_v(y_k)/v$

   We can start by guessing, for example, that $\theta = 0.8$ and $d_v(y_k)/v = 1/N_y$ for all $k =$

$1, ..., N_y$.

2. Guess bargaining solution for each pair $w(a_i, x_j, y_k)$.

   We can start from $w(a, x, y) = \gamma f(x, y)$, a fraction of the flow profit.

   Solve the worker's problem using the implicit method as in Achdou et al. (2020) (see Appendix B for details).

3. Calculate the stationary distribution of workers.

   Discretize the Kolmogorov forward (KF) equation as

$$0 = -\frac{s_{i,F}^{jk,W+} d_{i,}^{jk,W} - s_{i-1,F}^{jk,W+} d_{i-1}^{jk,W}}{\Delta_a} - \frac{s_{i+1,B}^{jk,W-} d_{i+1}^{jk,W} - s_{i,B}^{jk,W+} d_{i}^{jk,W}}{\Delta_a} - \delta d_i^{jk,W} + p(\theta) d_v(k) \mathbb{1}_i^{jk} d_i^{j,U}$$

$$0 = -\frac{s_{i,F}^{j,U+} d_{i}^{j,U} - s_{i-1,F}^{j,U+} d_{i-1}^{j,U}}{\Delta_a} - \frac{s_{i+1,B}^{j,U-} d_{i+1}^{j,U} - s_{i,B}^{j,U-} d_{i}^{j,U}}{\Delta_a} - p(\theta) \sum_k d_v(k) \mathbb{1}_i^{jk} d_i^{j,U} + \delta \sum_k d_i^{jk,W}$$

   We can write the system of KF equations compactly in matrix form:

$$\mathbf{A}(\mathbf{W}^n)' d = 0$$

   and the scale of the worker density is pinned down by the fact that d sums up to 1. $d_e(a, x, y)$ and $d_u(a, x)$. $d_v(y) = 1/N_y - \int d_e(a, x, y) \, \mathrm{d}a\mathrm{d}x$

4. Solve the firm's problem. The discretized HJB equation for firm is

$$\rho J_i^{jk} = f^{jk} - w_i^{jk} - \delta J_i^{jk} + s_i^{jk,W} J_{a,i}^{jk}$$

   Update the value function by

$$\frac{J^{l+1} - J^l}{\Delta} = f^{jk} - w_i^{jk} + \mathbf{A}_{1,e} J^{l+1} - (\rho + \delta) J^{l+1} \Rightarrow J^{l+1} = \left[\left(\frac{1}{\Delta} + \rho + \delta\right) - \mathbf{A}_{1,e}\right]^{-1} \left(f^{jk} - w_i^{jk} + \frac{1}{\Delta} J^l\right)$$

5. Update wage schedule according to the expression given by equation (15) in Appendix A.1.

6. update θ and dv, go back to 1.

7. Derive other densities and update guess in step 1.

Detailed algorithm is shown in Appendix B.

3 Theoretical Results

3.1 Two Limiting Economies

Our model provides a unified framework of incomplete market and frictional sorting. It is a generalization that nests Krusell, Mukoyama and Şahin (2010) and Shimer and Smith (2000).

If worker's preference is risk neutral, i.e., if the flow utility function u is linear in consumption c, then our model becomes the frictional sorting model a la Shimer and Smith (2000). Alternatively, if workers have access to a full set of Arrow securities (i.e., complete market), the model becomes Shimer and Smith (2000). In either case, asset level will not affect optimal decision and becomes irrelvant to decision making.

If $\mathbb{X}$ and $\mathbb{Y}$ are singletons, then we have homogenous workers (in terms of the skills; workers are still heterogenous with respect to wealth) and firms. There is no sorting so to speak. The model becomes a standard Bewley (1977); Huggett (1993); Aiyagari (1994) type incomplete market model with Diamond-Mortensen-Pissarides search frictions. This has been explored by Krusell, Mukoyama and Şahin (2010).

3.2 Wealth, Job Search and Wages

In this section we discuss several key implications from the model about the interactions between wealth, job search strategies and wages. These results will help us understand the mechanisms through which wealth affects labor market allocation and output, and how the model generates an endogenous joint distribution of wealth and wages.

Proposition 1. ***Precautionary Mismatch.*** *The matching set is wider for lower-asset holders. Fix worker type x and job type y. If a is the marginal wealth level such that the adjusted match surplus is exactly zero, then wealthier workers reject the match while poor workers accept the match. Formally, fix arbitrary x and y, if $\hat{S}(a,x,y)=0$, then $\hat{S}(a',x,y)<0$ for any $a'>a$, and $\hat{S}(a'',x,y)>0$ for any $a''<a$.*

Proof: See Appendix C.1.

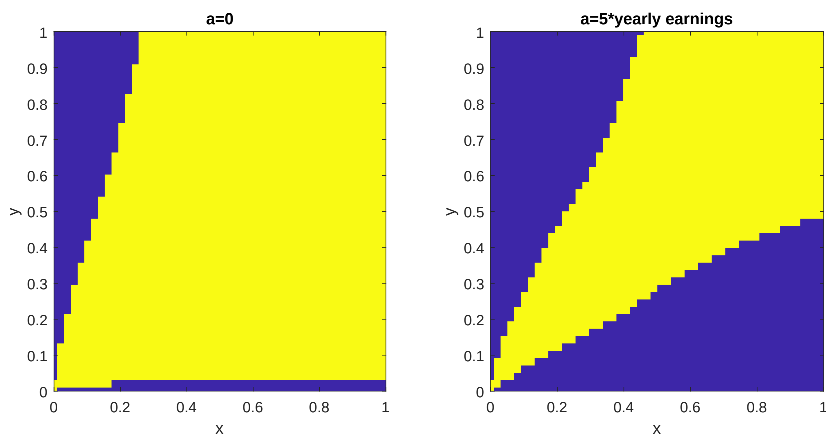
It is worth noting that this result does not rely on the properties of production function or sorting patterns. As long as workers have precautionary motives for self-insurance, matching sets will always be larger for lower-asset workers. In Figure 1, we demonstrate two cases of matching sets, depicted by the yellow areas. Panel (1a) shows matching sets when the production function is supermodular[2]. In this case, workers and firms are heterogeneous in terms of the level of skills/skill requirements (i.e. vertical heterogeneity), and the allocation features positive

[2] $f(x,y)=f_0+f_1\left(x^{\xi}+y^{\xi}\right)^{1/\xi}$, $0<\xi<1$.

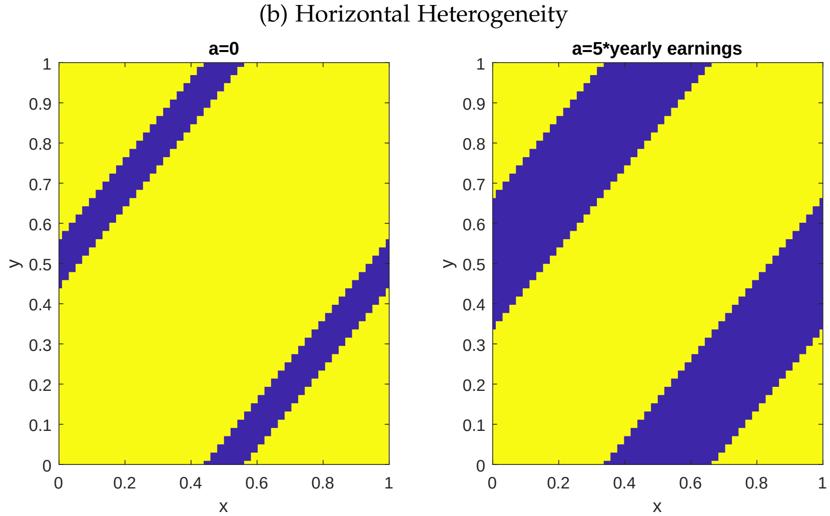
assortative matching (PAM) so that matches between workers (horizontal axis) and firms (vertical axis) happen along the 45-degree line. Panel (1b) shows the case where the production function is circular [3], which means that workers and firms are located on a unit-circle and are heterogeneous only in terms of the type of skills/skill requirements but not levels (i.e. horizontal heterogeneity). In this case, matches happen where worker and firm skill types are close to each other on the unit-circle. In both cases however, there is a range of acceptable matches around the perfect matches due to search friction. The left figures of each panel show the matching sets for workers with the lowest wealth in our model, and the right figures correspond to workers with assets worth of 5 times yearly earnings. Indeed, low-wealth workers have wider matching sets regardless of the property of the matching functions.

Figure 1: Matching Sets

(a) Vertical Heterogeneity



(b) Horizontal Heterogeneity



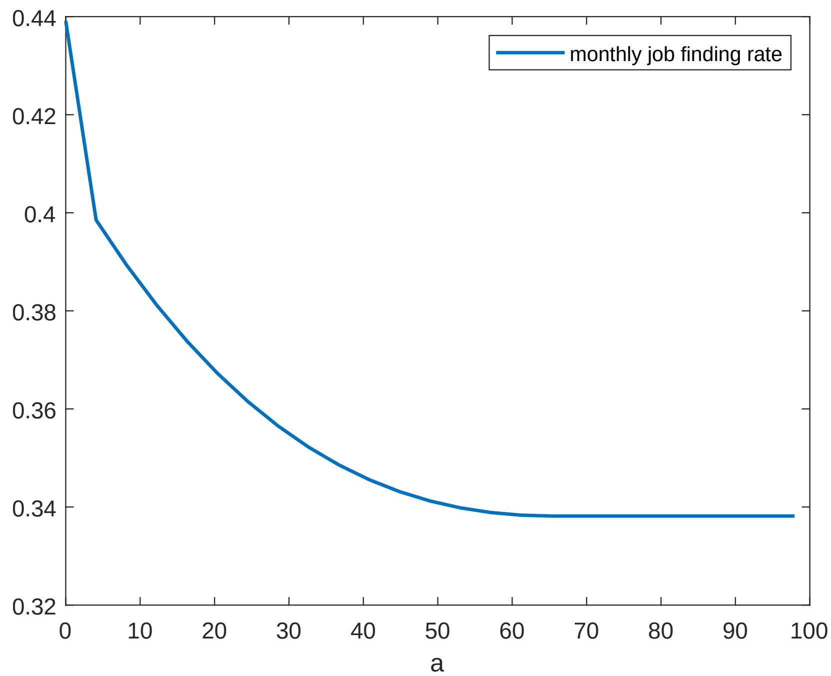
[3] $f(x, y) = a - b \min\left(|x-y|, |1+x-y|, |1+y-x|\right)^2$.

Proposition 2. ***Wealth and Job Finding Rate.*** *Job finding rate $\pi_{ue}(a, x)$ is decreasing with respect to wealth a.*

Proof: See Appendix C.2.

Figure 2 shows the model-generated relationship between job finding rate and wealth levels under the case of horizontal heterogeneity[4]. Since low-asset workers are more eager to find a job and accept a wider range of meetings, their job finding rate is necessarily higher. This finding has been documented empirically by Lise (2013) using NLSY79 data.

Figure 2: Job Finding Rate by Wealth



Proposition 3. ***Wealth and Wages.*** *Average wages $\bar{w}(a, x) = \int w(a, x, y)\,\Phi(a, x, y)\,\frac{d_v(y)}{y}dy$ is increasing in wealth a.*

We provide some intuitions rather than a formal proof for this result, and for conciseness we focus on the case with horizontal heterogeneity. Note, however, that the result is also true under other sorting patterns. In Figure 3, we plot the wage functions under a perfect match $(x = y)$ in blue lines and a marginal match in red lines, which occurs at the edge of matching sets. For a marginal match, the worker and the firm are indifferent between accepting and staying unemployed/vacant.

[4]The relationship is conceptually the same under vertical heterogeneity, but job finding rate in this case would also depend on worker skill levels.

The top panel of Figure 3 shows that wages for both the perfect match and the marginal match increases with wealth. For perfect match, wage increases with wealth for the same reason as in Krusell, Mukoyama and Şahin (2010): the outside option, namely the value of being unemployed, increases quickly with wealth as precautionary motive dissipates, so that wealthier workers bargain for higher wages. For marginal match, wage increases for two reasons, as shown in the bottom panel of Figure 3. First, for any given match (purple or yellow line), wage increases with wealth due to Nash bargaining. Second, as workers become wealthier, their matching set shrinks, thus the marginal match gets closer to the perfect match. Therefore as wealth increases, wage not only increases along the marginal match wage curve but also across different marginal matches (the red dots correspond to points on the wage curves of different marginal matches). Since average wages lie in between the wage functions of the perfect match and the marginal match, we can conclude that average wages must also increase with wealth.

Proposition 4. ***Optimal Consumption Growth.*** *The Euler equations, which specify the optimal consumption growth paths for employed and unemployed workers, can be written as follows:*

$$\frac{\dot{c}^e}{c^e} = \frac{1}{\gamma}\left\{r - \rho + \omega_a + \sigma\left[\frac{u'(c^u)}{u'(c^e)} - 1\right]\right\}$$
$$\frac{\dot{c}^u}{c^u} = \frac{1}{\gamma}\left\{r - \rho - p(\theta)\int_{B(a,x)} \frac{d_v(y)}{v}\left[1 - \frac{u'(c^e)}{u'(c^u)}\right]\mathrm{d}y\right\}$$

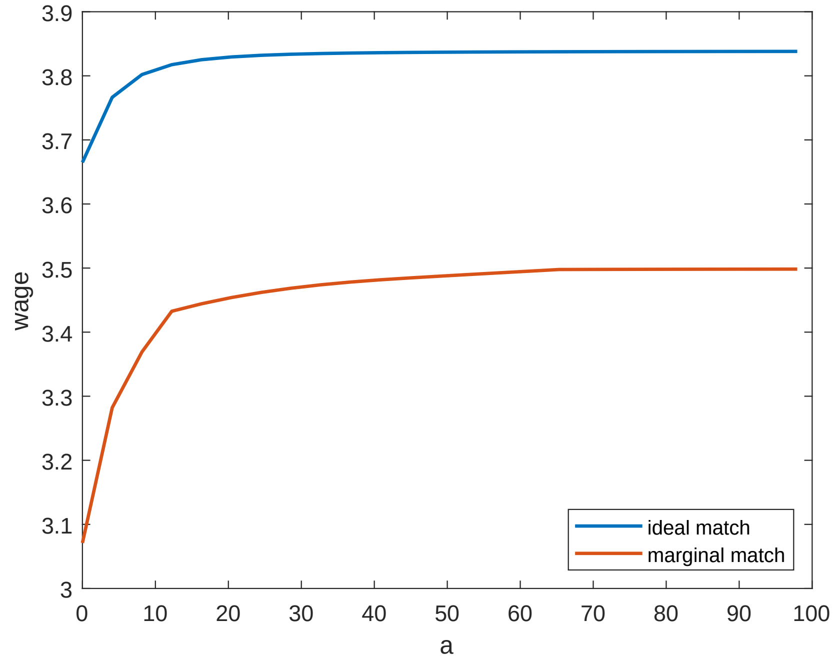
where arguments of $\omega(a,x,y), c^u(a,x,y), c^e(a,x,y)$ are suppressed for brevity. $B(a,x) := \{y : \Phi(a,x,y) = 1\}$ is the acceptance set of worker (a,x).

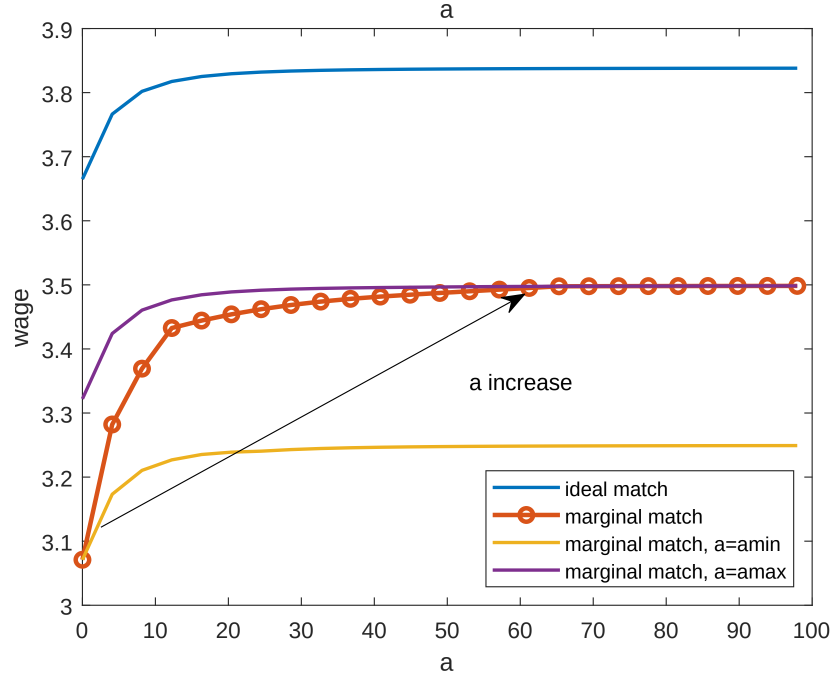
Proof: See Appendix C.3.

The equations show us the reasons behind workers' saving decisions. First, there is a standard saving motive due to the difference in interest rate r and the rate of time preference ρ. For employed workers, there are additional saving motives due to the effect of wealth on wages w_a, and the possibility of job loss. The term in square bracket corresponds to precautionary savings motive, which is particularly strong when wealth is low or current wages are high. For unemployed workers, there is a dis-saving motive due to the possibility of finding a job. Notably, the dis-saving motive depends on the worker's acceptance set $B(a,x)$. Everything else equal, a larger acceptance set induces unemployed workers to dis-save more. This term would be absent in models without endogenous job-finding strategies and two-sided heterogeneity.

These propositions show the reasons why our model generates an endogenous joint distribution of wealth and wages. Wealth affect wages by increasing workers' outside options and allowing job-searching workers to wait for better matches, while wages affect wealth by affecting the flow of income and saving rates. The joint distribution would be absent or degenerate in most existing models.

Figure 3: Wealth and Wages





4 Data

Our empirical analysis is based on a selected worker panel from the 1979 National Longitudinal Survey of Youth (NLSY79), a nationally representative survey conducted on individuals 14-22 years old when first interviewed in 1979. We merge the NLSY79 work history and asset information with data from the Occupational Information Network (O*NET), an occupation-level data set with scores on the skill contents of 974 occupations, so that we have a matched worker-job data set with joint worker and job characteristics. In the sections below we provide a description of the data sources, how the measures of worker skills (x) and job skill requirements (y) are constructed, as well as some sample statistics.

4.1 Data Sources and Skill Measures

NLSY79

We use the work history data from NLSY79 to construct a monthly panel, and focus on a cross-sectional sample of workers with no experience of serving in the military. We further exclude individuals who are already considered to be in the labor market at the beginning of the survey, where we consider an individual to be in the labor market if they work more than 30 hours per week or 1200 hours per year, or if they have finished their last schooling spell and started working. To minimize the effect of work experience gained during education on our estimation, we also exclude those who have more than 2 years of work experience before the end of his/her schooling spell.

Worker skill measures (x) are constructed using individual characteristics extracted from the test scores of the Armed Services Vocational Aptitude Battery (ASVAB), a special survey conducted by the US Departments of Defense and Military Services in 1980 that evaluates individuals in 10 categories. As the test was conducted before the majority of the respondents entered the labor market, we believe that our skill measure is mostly free from the endogeneity issue wherein jobs affect worker skills.

To construct skill bundles for workers, we follow a procedure similar to the one used in Lise and Postel-Vinay (2020) (see Appendix D.1 for details): we run principal component analysis (PCA) on the 10 individual-level ASVAB test scores and keep the first two principal components. We then construct worker skills along 2 dimensions, namely cognitive and manual skills by recombining the principal components so that they satisfy the following exclusion restrictions: (1) the ASVAB mathematicas knowledge score only loads on cognitive skill, and (2) the ASVAB automotive and shop information score only loads on manual skill. We then take the percentile ranks of the two principal components, so that the worker skill measures are distributed on a unit-length interval $[0, 1]$.

In addition to work history and test scores, we also obtain annual history on assets from NLSY79. Unfortunately, NLSY79 did not start extensively collecting assets information until 1985, when over half of the respondents had entered the labor market. Therefore our sample is heavily biased towards late-entrants when examining the relationship between liquid wealth and skill mismatch. We construct a measure of liquid wealth of individuals based on the sum of financial assets such as cash, deposit, mutual fund and money market accounts and other assets more than $500, net of debts that are not asset-backed. Since asset information is not updated in each round of survey for most respondents, we linearly interpolate the amount of assets for each individual to maximize the amount of information we can use in our empirical analysis.

O*NET

The O*NET data contains ratings of importance and level on hundreds of specific aspects, called "descriptors", of each occupation. The descriptors can be summarized by 9 broad categories: skills, knowledge, abilities, work activities, work context, education levels required, job interests, work styles and work values. Following Lise and Postel-Vinay (2020), we keep the level ratings related to descriptors from the first 6 categories, which add up to over 200 descriptors for each occupation.

Similar to the procedure for worker skills construction, we reduce the descriptors to 2 dimensions using PCA and keep the first 2 components. Then, we recover cognitive and manual skill requirements by recombining the principal components in such a way that (1) the mathematics rating only loads on cognitive skill requirements, and (2) the mechanical knowledge rating only loads on manual skill requirements. We take the percentile ranks of the two principal components so that job skill requirements lie on a unit-length interval $[0, 1]$. Therefore, each job can be characterized by a bundle of skill requirements (y), in which a higher number in each dimension represents higher requirements of the corresponding skill.

For this paper, however, we only focus on sorting based on cognitive skills. While our structural model can easily account for multidimensional skill types in theory, solving and estimating such a model turns out to be computationally heavy.

4.2 Descriptive Statistics

4.2.1 Skill Measures and Sorting

Our selected sample includes 3,285 individuals with substantial heterogeneity in levels of education, ranging from no degree to PhD. Presumably, our measure of worker skills and job skill requirements should reflect their relative rankings and productivity in the sample respectively. An obvious way to examine this presumption is to see how the two measures vary by levels of education. Table 2 shows average worker cognitive skills and job cognitive skill requirements by highest degrees at the time of initial labor market entry. Both measures are normalized to a unit-length range $[0, 1]$, where a higher number represents higher cognitive skill/skill requirement.

Table 2: Average Worker Cognitive Skills and Job Skill Requirements, by Level of Education

	No Degree	High School	Some College	2-yr College	4-yr College	Masters	PhD
Worker Skill (x)	0.266	0.388	0.456	0.543	0.684	0.714	0.770
Job Skill Req (y)	0.223	0.272	0.304	0.336	0.425	0.474	0.504
Observations	159	1053	233	402	500	323	99

Note: Both x and y are normalized to $[0, 1]$.

Comparison of the skill measures at the lowest and highest education levels (No Degree and PhD) shows that education seems to account for a substantial amount of worker skill heterogeneity, and a modest amount of job skill heterogeneity. It is perhaps not surprising to find that both worker skills (first row) and job skill requirements (second row) increase monotonically with level of education. Therefore at least with respect to skill differences across education groups, our skill measures are able to capture the relative ranking of workers and jobs, as well as positive sorting. However, some questions yet to be answered are whether we can identify sorting beyond sorting on education using the cognitive skill measures, and whether sorting is still positive after controlling for education groups. To answer this question, we show the correlation between job skill requirements and worker skills in Table 3, with and without controlling for worker educations.

We take one observation from each worker-employer match in the data and regress job cognitive skill requirements on worker cognitive skills. Column (1) shows that the correlation between worker skills and job skill requirements are 0.69, which is both statistically and economically significant. To isolate sorting on skills from sorting on education, we perform an additional

Table 3: Skill Sorting Over Occupations

	(1) Job Skill Req (y)	(2) Job Skill Req (y)
Worker Skill (x)	0.691***	0.513***
	(0.020)	(0.027)
Education Level	No	Yes
Obs	35616	35616

Note: Standard errors are clustered on occupation level. The sample is taken from the first observations of each worker-employer match.

regression controlling for dummies for years of education. After controlling for education, the remaining correlation is still large and highly significant at 0.51, suggesting a substantial amount of sorting on individual skills exists beyond sorting on education.

4.2.2 Initial Liquid Wealth and Worker Characteristics

There are 1,114 individuals with valid information about liquid financial wealth at the time when they entered labor market. We define net liquid wealth as the value of financial assets such as cash, deposit, mutual fund and money market accounts net of debts that are not asset-backed. This measure is supposed to reflect assets that workers can access in a relatively short period of time. Table 4 shows the characteristics of workers upon labor market entry, where the workers are divided into quintiles according to their liquid wealth during the first month of work.

There are substantial heterogeneity in the level of liquid wealth upon labor market entry, ranging from $-7,971 in the lowest quintile to $31,830 in the highest quintile (in 1982 dollars), a difference of almost $40,000. Workers who enter the labor market with higher liquid wealth tend to have higher income, more education, higher age and higher parental income. The only exception is the lowest quintile, where weekly income, years of education, age and parental income are all higher than those in the quintile above. A likely explanation is that the lowest liquid wealth quintile could consist of individuals who borrow substantial amount of debt for their higher education, thereby lowering their initial wealth. Note that the age of labor market entry is highly upward biased (most workers enter labor market in early 20s) because NLSY79 didn't start collecting wealth information until 1985, when half of the sample were above 25. This means that later when we analyze the effect of initial wealth, our sample is biased towards late

Table 4: Worker Characteristics by Initial Wealth Quintile

	Quintile 1	Quintile 2	Quintile 3	Quintile 4	Quintile 5
Net Financial Assets (1000s)	-7.971	0.346	1.764	5.356	31.83
Weekly Income	233.8	195.2	193.3	255.8	301.4
Years of Educ	15.91	14.54	15.38	15.88	16.25
Age	27.48	27.09	26.98	27.68	29.13
Male	0.416	0.405	0.368	0.446	0.350
PRTs Annual Income	19874.5	18343.5	23147.3	25479.3	25623.4
Observations	202	200	204	202	203

Note: liquid assets, weekly income and parents' annual income are in 1982 dollars

entrants.

5 Empirical Analysis

In Section 3.2 we discussed several implications about wealth, job search and wages generated by the model. Now we use the merged NLSY79 and O*NET data to examine whether these implications are supported by empirical evidence.

5.1 Precautionary Mismatch

First, let us provide a formal definition of mismatch used for our empirical analysis.

Definition 1. Mismatch measures

*Let x_i denote the skill level of individual i, and y_j denote the skill requirement of job j, then we define the **mismatch** between individual i and job j as*

$$m_{i,j} \equiv y_j - x_i \tag{12}$$

$m_{i,j} > 0$ *means that worker i is under-qualified (or over-employed) for job j, and vice versa. We define the* *magnitude of mismatch* *between individual i and job j as*

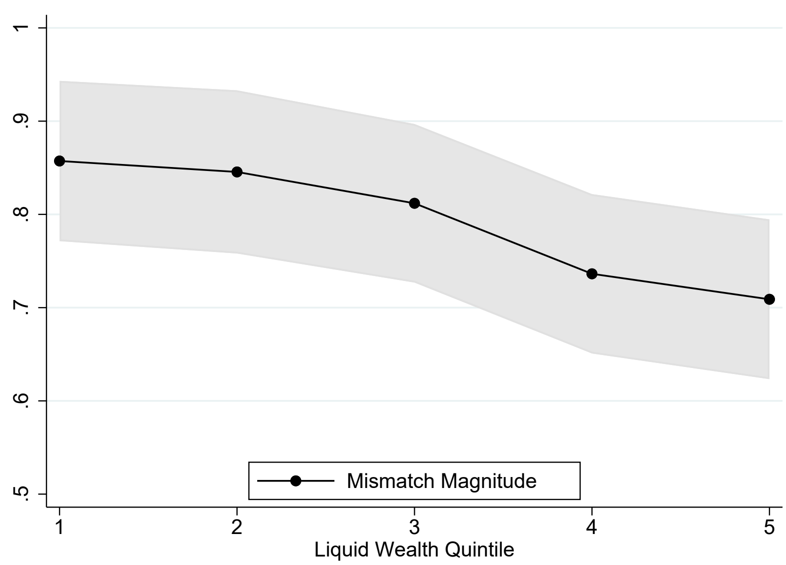
$$mm_{i,j} = |m_{i,j}| \tag{13}$$

We normalize mismatch $m_{i,j}$ so that its average is 0. By doing so we implicitly assume that in aggregate, there is as much over-qualification (i.e. $m_{i,j} < 0$) as under-qualification ($m_{i,j} > 0$) in the labor market. We also re-scale the levels so that the mismatch measure has a unit standard deviation.

Wealth and Mismatch

We now document the relationship between wealth and skill mismatch, which was discussed in Proposition 1. As we know from Table 4, initial wealth might be confounded by workers' levels of education, which could in turn also affect sorting. Therefore we control for level-of-education fixed effects when plotting the levels of mismatch. Figure 4 shows the relationship between liquid wealth and mismatch for labor market entrants.

Figure 4: Mismatch Measures by Wealth Quintile



We can see from of Figure 4 a clear drop in the magnitude of skill mismatch $|y - x|$ with liquid wealth, from about 0.85 in the lowest wealth quintile to around 0.7 in the highest wealth quintile. This shows that for wealthier workers, the set of acceptable jobs are smaller, leading to lower levels of mismatch. This finding is consistent with the theoretical results from Proposition 1.

5.2 Wealth and Job Finding Rate

Next, we examine empirically whether the relationship between wealth and job finding rate, discussed in Proposition 2 of Section 3.2, also holds in the data. Table 5 shows regression estimates where we regress the log of workers' monthly job finding rates on wealth positions[5] at labor market entry, along with other covariates.

Table 5: Job Finding Rate and Wealth

	(1)	(2)	(3)
Log Net Liquid Wealth	-0.005	-0.007**	-0.010***
	(0.003)	(0.003)	(0.004)
Demographic	No	Yes	Yes
Family background	No	No	Yes
Obs	5368	5368	4264

Column (1) of Table 5 shows the coefficient of liquid wealth with it being the only regressor. In column (2), we control for a standard set of individual characteristics including sex, race, education, age and AFQT score (a proxy for ability). To further control for insurance that young workers may receive from their families, we additionally include parents' annual income and poverty status in column (3). The estimates imply that a 1 percent increase in liquid wealth is associated with a 0.01 percent decrease in monthly job finding rate, which is qualitatively consistent with Proposition 2 and also similar in magnitude to the findings by Lise (2013).

5.3 Wealth and Wages

Here we revisit Proposition 3 of Section 3.2 and check whether the model-implied relationship between wealth and wages holds in the data. Table 6 shows the coefficients of log-wage regressions where the regressors include logged net wealth and a set of control variables.

The coefficients on log net worth are all highly significant, and suggest that a 1 percent increase in net worth is related to a 0.02 percent increase in wages. This suggests that the amount of wage dispersion created by wealth dispersion should be positive but small, which is supported by our model as well as Krusell, Mukoyama and Şahin (2010).

[5] For all regressions involving wealth, I follow Lise (2013) and use the inverse hyperbolic sine transformation of wealth, $\log\left(a + \sqrt{1+a^2}\right)$.

Table 6: Wages and Wealth

	(1)	(2)	(3)
Log Net Liquid Wealth	0.024***	0.015**	0.014**
	(0.007)	(0.006)	(0.007)
Demographic	No	Yes	Yes
Family background	No	No	Yes
Obs	3189	3189	2515

Mismatch and Wages

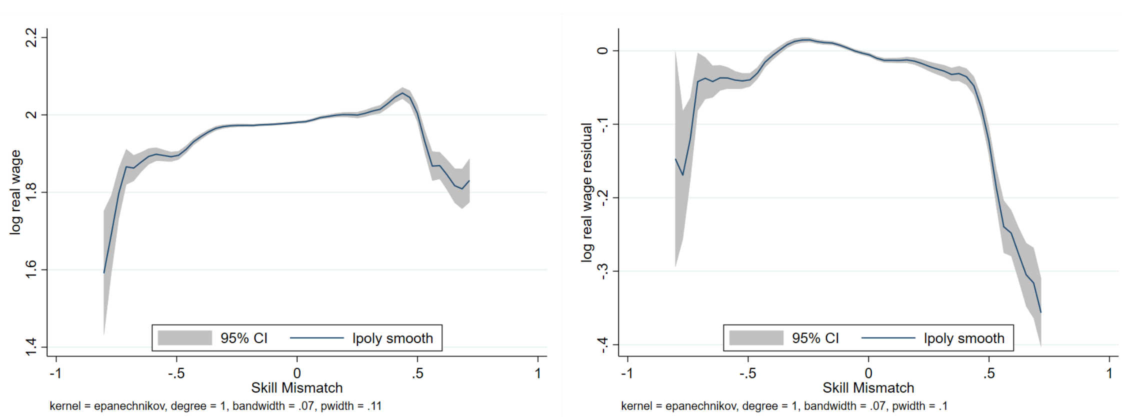
Proposition 3 of Section 3.2 states that wealth affects wages through two channels: Nash bargaining, which allows wealthier workers to bargain for higher wages, and a decrease in mismatch, which allows matches to be more productive. Now we provide some suggestive evidence that the mismatch channel is consistent with data.

Figure 5 shows non-parametric plots of log wages (in 1982 dollars) as a function of the deviation from job's skill requirement $(y - x)$, based on kernel smoothed local linear regressions. The left panel is based on raw wage data, while the right panel is based on residual wages by estimating the following wage regression

$$\ln w_{i,l,c,t} = X_{i,l,c,t}\beta + \epsilon_{i,l,c,t}$$

where $w_{i,l,c,t}$ is real wage of an individual i working with employer l in occupation c in period t and $\epsilon_{i,l,c,t}$ is the residual. The control variables in X includes race, sex, education fixed effects, quadratic functions of employer tenure, occupation tenure, labor market experience and age as well as 3-digit occupational fixed effect.

Figure 5: Log Wages by Skill Mismatch



While the scales in the two plots are not directly comparable because the right panel uses wage residuals, we can see that in both cases, wages tend to be higher when mismatch is close to 0, and lower when job skill requirement is either too high or too low relative to the worker's skill. This figure provides evidence for the aforementioned theories and suggests that skill mismatch is directly linked with returns to worker skills.

6 Numerical Exercises

Having shown that our model's key predictions are consistent with data, we now turn our focus back to the model and discuss how we plan to provide quantitative results.

6.1 Parameterization

We adopt standard functional form assumptions to facilitate numerical analysis. We assume the flow utility function exhibits constant relative risk aversion (CRRA):

$$u(c) = \frac{c^{1-\gamma}}{1-\gamma} \quad , \quad \gamma > 0.$$

The meeting function is assumed to take the Cobb-Douglas form:

$$M(u, v) = \chi u^{\alpha} v^{1-\alpha}.$$

Without loss of generality, worker and job types are normalized to be uniformly distributed. To see its generality, suppose the $\tilde{F}(\tilde{x})$ and $\tilde{G}(\tilde{y})$ are the cumulative density functions of the distribution of worker and job types, respectively, with a production function $\tilde{f}(\tilde{x}, \tilde{y})$. We could redefine a type according to its rank, i.e., $x := \tilde{F}(\tilde{x})$ and $y := \tilde{G}(\tilde{y})$, and rewrite the production function accordingly $f(x, y) := \tilde{f}\left(\tilde{F}^{-1}(x), G^{-1}(y)\right)$. The distribution of the rank-based type is thus uniform, as the CDF of any random variable is uniformly distributed between 0 and 1.[6] We specify a production function that induces positive assortative matching (PAM):

$$f(x, y) = f_0 + f_1 \left(x^{\xi} + y^{\xi}\right)^{1/\xi}, 0 < \xi < 1 \tag{14}$$

ξ controls the degree of complementarily between worker skills x and job skill requirements y.

[6]To see this, denote the transformed cumulative distribution functions as F and G such that $x \sim F$ and $y \sim G$. Consider an arbitrary $t \in [0, 1]$. We have

$$F(t) = \mathbb{P}(x \le t) = \mathbb{P}\left(\tilde{F}(\tilde{x}) \le t\right) = \mathbb{P}\left(\tilde{x} \le s, \text{ for some } s \in \tilde{F}^{-1}(t)\right) = t.$$

Therefore $x \sim \mathcal{U}[0, 1]$. Similarly, $y \sim \mathcal{U}[0, 1]$.

A less positive ξ leads to stronger complementarity. Empirical evidence in Hagedorn, Law and Manovskii (2017) supports PAM as a description of data. It is also intuitively correct to allow different workers and firms to have different levels of skills/skill requirements.

6.2 Calibration

Note: This part is still under construction.

We assume the borrowing constraint is $\underline{a} = 0$ such that workers cannot have negative net worth. For the numerical exercise, we use grids with 50 worker types and 50 occupations. We assume that in the stationary equilibrium, the vacancy posting cost is such that there is the same number of jobs as workers.

Our model is calibrated to match aggregate U.S. data on a quarterly frequency. Table 7 summarizes the parameter values used in our numerical exercise. Some parameters are set as standard values in the literature, while others are calibrated internally in the model. In particular, we calibrate the PAM production function to match moments of wage dispersion in the data. We set the level of home production b as 40 percent of the model-generated minimum wage. This is different from standard practices in the literature where b equals 40 percent of average wages. We make this choice due to vertical heterogeneity in worker skills, which generates a substantial gap between wages of high-skilled and low-skilled workers. The home production level we choose prevents low-skilled workers from having too strong a dis-incentive to work.

Note that although Hosios condition is imposed, it does guarantee efficiency here.

6.3 Precautionary Mismatch and Labor Productivity

Once we fully calibrate the model, which will be incorporated in our future iterations, we can try to answer the question we asked at the beginning: how does wealth affect labor market allocation and aggregate labor productivity? We approach this question in two steps. First, we can estimate how labor misallocation in general affects labor productivity. To do so, we take the stationary-equilibrium labor market allocation as baseline and estimate by how much the labor market could be more efficient if we allocate workers to firms in a way that maximizes output. Second, we can examine how sensitive the allocation and thus aggregate productivity is to wealth distribution. For this exercise we start with the stationary equilibrium and hit the economy with an unexpected wealth shock which reduces every worker's wealth by 50%. We can then see how labor allocation changes from the stationary equilibrium on impact.

Table 7: Calibration

Parameter	Value	Source
	External Calibration	
interest rate	$r = 0.01$	annual interest rate 4%
relative risk aversion	$\gamma = 2$	common parameterization
bargaining power	$\eta = 0.72$	Shimer (2005)
matching elasticity	$\alpha = 0.72$	Hosios condition
separation rate	$\sigma = 0.1$	monthly job losing rate 0.034
	Internal Calibration	
discount rate	$\rho = 0.011$	capital-output ratio
home production	$b = 0.02$	40% min wage
production function	PAM	Gini & std. dev. of wages
matching efficiency	$\chi = 3$	monthly job finding rate 0.45

Notes: The table summarizes the calibrated parameters and their sources.

6.3.1 The Effect of Mismatch on Labor Productivity

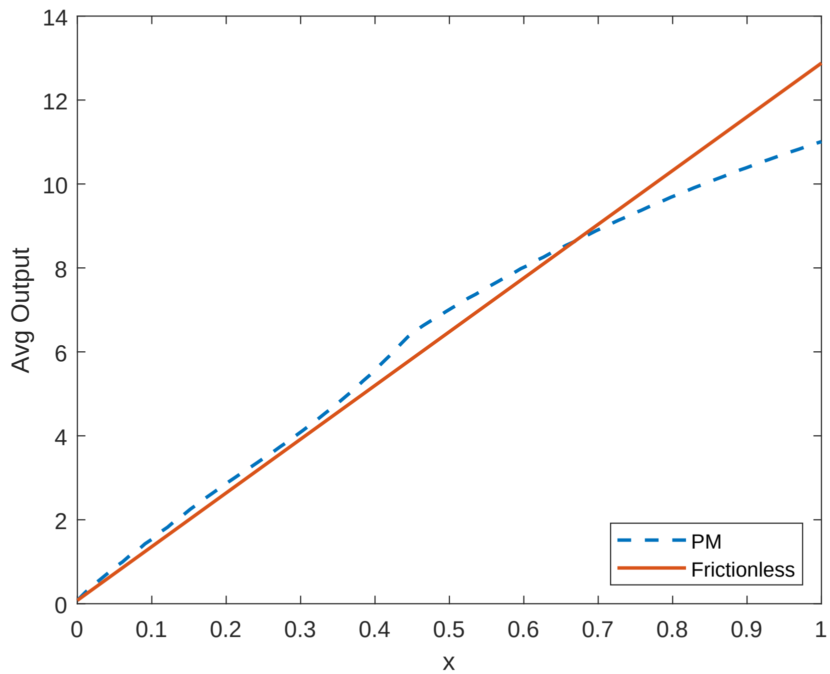
In the first exercise, we compare the allocation in the model's stationary equilibrium with a counterfactual allocation in an economy without search frictions. In the counterfactual economy, there is a function μ that maps workers to firms $\mu : \mathcal{A} \times [0,1] \to [0,1]$ so that output is maximized. Let Y^{OPT} be the total output of the frictionless economy, and Y^{PM} be the output of the stationary equilibrium with precautionary mismatch, then

$$Y^{OPT} = \int_0^1 \int_{\mathcal{A}} f\left(x, \mu\left(a, x\right)\right) \mathrm{d}a\mathrm{d}x$$
$$Y^{PM} = \int_0^1 \int_0^1 \int_{\mathcal{A}} f\left(x, y\right) d_m\left(a, x, y\right) \mathrm{d}a\mathrm{d}x\mathrm{d}y$$

and since the measure of workers is normalized to 1, aggregate productivity of the frictionless economy is just Y^{OPT} and that of the stationary equilibrium is $Y^{PM} / \left(1 - u\right)$, where u is the measure of unemployed workers.

Since the production function used in the numerical exercise induces PAM, the assignment rule in the frictionless economy is $\mu\left(x\right) = x$, i.e. each worker is allocated to a firm where the skill requirement is equal to her skill level. Because of vertical skill heterogeneity, it is also useful to compare productivity levels of different worker types under the two scenarios. Figure 6 shows average output per worker by worker type x. The blue dashed line ("PM") refers to the economy under stationary equilibrium with precautionary mismatch, while the solid red line ("Frictionless") corresponds to the frictionless economy.

Figure 6: Productivity by Worker Type



While the allocation in the frictional economy necessarily results in lower aggregate productivity than that in the frictionless economy, as the assignment rule in the frictionless economy is output-maximizing by definition, an interesting pattern emerges from the comparison. When the economy is frictional, high-skilled workers lose productivity, while low-skilled ones actually gain productivity. This is because mismatch provides a possibility for low-skilled workers to be matched with relatively high productivity jobs, while also making it harder for high-skilled workers to find their best match. Therefore with vertical heterogeneity, a match that is good from an individual worker's perspective is not always good from the economy's perspective: high productivity jobs would produce much more if matched instead with high-skilled workers.

Note that this comparison only provides a statement about the productivity effect of search frictions, but not the precautionary mismatch motive (which is in turn a result of interactions between wealth and search frictions). If anything, Figure 1 in Section 3.2 suggests that low-wealth workers tend to find jobs with lower requirements, which depresses their potential productivity. In the next exercise, we highlight the role of wealth and thus precautionary mismatch motive in shaping labor productivity.

6.3.2 The Effect of Wealth on Labor Productivity

In the second exercise, we hit the stationary-equilibrium economy with a financial shock which lowers each worker's wealth by 50%. Upon impact, the distributions of employed and unemployed workers with previous wealth level a become $\hat{d}_m(a,x,y) = d_m\left(\frac{a}{2},x,y\right)$ and $\hat{d}_u(a,x) = d_u\left(\frac{a}{2},x\right)$. In addition, their job acceptance strategy becomes $\hat{\Phi}(a,x,y) = \Phi\left(\frac{a}{2},x,y\right)$. We therefore solve for a new measure of worker-firm matches $\hat{d}(x,y)$ that satisfies

$$\hat{d}(x,y) = \int_{\mathcal{A}} \hat{d}_u(a,x)\, p(\theta)\, \frac{d_v(y)}{v} \hat{\Phi}(a,x,y) + (1-\sigma)\, \hat{d}_m(a,x,y)\, \mathrm{d}a$$

where the integral is over original wealth level a. This is the measure of worker-firm matches at the moment when the shock hits and workers adjust their acceptance sets. With the measure we can compute total output after the financial shock Y^{FIN}:

$$Y^{FIN} = \int_0^1 \int_0^1 f(x,y)\, \hat{d}(x,y)\, \mathrm{d}x\mathrm{d}y$$

and measure of employed workers

$$\hat{u} = \int_0^1 \int_0^1 \hat{d}(x,y)\, \mathrm{d}x\mathrm{d}y$$

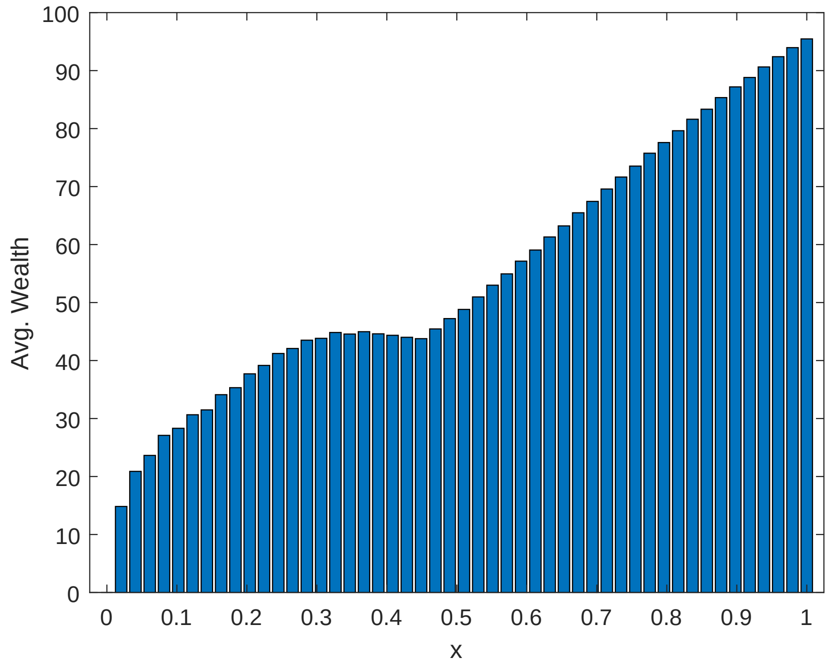
which gives aggregate labor productivity $Y^{FIN}/\hat{u}$.

Note that since wealth affects the incentive for workers and firms to accept matches, allocations of workers to firms will change in response to a wealth shock. In addition, in the original equilibrium there is an endogenous correlation between worker skill levels and wealth, so that different types of workers are affected differently by the shock. Figure 7 shows average wealth held by workers of different skills in the stationary equilibrium. As high-skilled workers receive higher wages, they also enjoy higher wealth on average. When the shock hits, they are the group that suffers the heaviest wealth decline.

For the sake of comparison with the first example, we plot again average productivity by worker type x. In Figure 8, the blue dash line and red solid line are the same as in Figure 6, while the yellow dash-dotted line corresponds to the economy after the financial shock.

Comparing the economies before (blue dash line) and after (yellow dash-dotted line) the shock, we see a uniform decrease in productivity levels, especially for high-skilled workers. This is in large part due to the positive correlation between skill and wealth that we mentioned before: since high-skilled workers suffer the most wealth decline, they experience a significant increase in precautionary mismatch motive. Additionally, these workers have the strongest complementarity

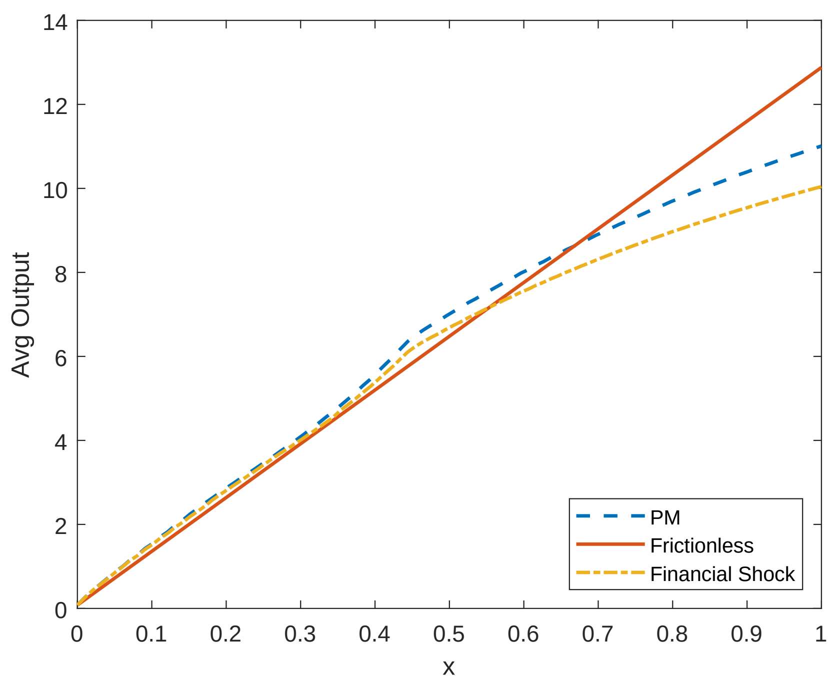
Figure 7: Average Wealth by x in the Stationary Equilibrium



with high-skill-requirement jobs, which makes it particularly costly for them to be mismatched in terms of output. Consequently, the shock exacerbates the negative effect of precautionary mismatch on labor productivity.

While in principle we can calculate aggregate productivity loss due to the financial shock, we think it is more useful to provide a quantitative result after the model is fully calibrated.

Figure 8: Productivity by Worker Type



7 Conclusion

The goal of our paper is to answer the question: how does wealth affect the allocation of workers to firms, and to what extent can labor market efficiency be affected by wealth distribution? We develop a framework with two-sided heterogeneity, search frictions and incomplete markets, where workers need to adjust their job acceptance strategies to self-insure against unemployment shocks. We find both theoretically and empirically that under precautionary motive, wealth-poor workers speed up job search by accepting higher degrees of skill mismatch, at the cost of potentially lower wages. The precautionary mismatch motive thus weakens sorting, and leads to more mismatched pairs of workers and jobs.

We highlight the role that wealth plays in affecting labor productivity using a counterfactual exercise in which 50% of wealth is erased from all workers. Stronger precautionary motive induces workers to accept higher degrees of skill mismatch, which depresses their productivity. The effect is particularly pronounced for high-skilled workers, as they tend to suffer the most wealth decline and their mismatches are more costly due to strong complementarity with job skill requirements.

While we refrain from providing quantitative results on how sensitive aggregate productivity is to wealth distribution due to the ongoing nature of calibration, we would like to point out that a fully calibrated version of our model is well-equipped to answer this fascinating question, and can also serve as a tool to analyze the role of social insurance policies such as UI in affecting labor market efficiency. Intuitively, UI serves as a buffer for low-wealth workers as it prevents extreme consumption fluctuations under unemployment shocks, which would lower their precautionary mismatch motive, thereby increasing allocative efficiency in the labor market. By stressing the allocation of heterogeneous workers to heterogeneous firms, our model provides a new perspective on how UI affects our economy, in addition to the standard trade-off between insurance and employment incentive discussed in the literature.

Last but not least, since our model features an endogenous joint distribution of wealth and wages, we provide future works with a tool to study the interactions between wage and wealth inequality. In our model, wealth affect wages by allowing workers to bargain for higher wages and wait for better matches, and wages affect wealth by affecting income flows as well as saving rate. While the baseline model is unlikely to capture the entire wealth distribution that we see in the data, many theories, such as entrepreneurship, preference heterogeneity and return heterogeneity, have been proposed to match the observed wealth dispersion and are very successful in achieving this goal. It might be challenging yet promising to augment our current model with these elements to provide a unified theory of wage and wealth inequality.

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A Mathematical Appendix

A.1 Nash Bargaining

To derive the wage setting, we start with the discrete time problem with period length of Δ. The value for employed worker of type x with asset a that works at a job of type y for an arbitrarily deviating flow wage w (recognizing that in the following period the wage will go back to the equilibrium bargained wage) satisfies

$$\begin{aligned}\tilde{W}(w,a,x,y) &= \max_c u\left(c\right)\Delta + \frac{1}{1+\rho\Delta}\left\{\left(1-\Delta\sigma\right)W\big(a',x,y\big) + \Delta\sigma U\left(a',x\right)\right\}\\ \text{s.t.}\quad & a' = a + \left(ra + w - c\right)\Delta.\end{aligned}$$

Denote the optimal consumption policy by $\tilde{c}^e\left(w,a,x,y\right)$. The value function could be written as

$$\tilde{W}(w,a,x,y) = u\left(\tilde{c}^e\right)\Delta + \frac{1}{1+\rho\Delta}\left\{\left(1-\Delta\sigma\right)W(a+\left(ra+w-\tilde{c}^e\right)\Delta,x,y) + \Delta\sigma U\left(a+\left(ra+w-\tilde{c}^e\right)\Delta,x\right)\right\}.$$

Multiply both sides by $(1+\rho\Delta)$, subtract $\tilde{W}$, and then divide them by Δ,

$$\begin{aligned}\rho\tilde{W}(w,a,x,y) = {}& u\left(\tilde{c}^e\right)\left(1+\rho\Delta\right) + \frac{1}{\Delta}\left[W(a+\left(ra+w-\tilde{c}^e\right)\Delta,x,y) - \tilde{W}(w,a,x,y)\right]\\ &+ \sigma\left[U\left(a+\left(ra+w-\tilde{c}^e\right)\Delta,x\right) - W(a+\left(ra+w-\tilde{c}^e\right)\Delta,x,y)\right].\end{aligned}$$

Similarly, the value for such a producing job is

$$\tilde{J}(w,a,x,y) = f\left(x,y\right)\Delta - w\Delta + \frac{1}{1+\rho\Delta}\left[\left(1-\Delta\sigma\right)J\big(a',x,y\big) + \Delta\sigma V\left(y\right)\right],$$

where $a' = a + \left(ra + w - \tilde{c}^e\left(w,a,x,y\right)\right)\Delta$ is taken as given from the firm's point of view. Using a similar procedure (i.e., multiply both sides by $1+\rho\Delta$, subtract $\tilde{J}$, and then divide them by Δ), we obtain

$$\rho\tilde{J}(w,a,x,y) = f\left(x,y\right)\left(1+\rho\Delta\right) - w\left(1+\rho\Delta\right) + \frac{1}{\Delta}\left[J\big(a',x,y\big) - \tilde{J}(w,a,x,y)\right] + \sigma\left[V\left(y\right) - J\big(a',x,y\big)\right].$$

Under Nash bargaining, the wage policy is determined by

$$\omega\left(a,x,y\right) = \arg\max_w \left[\tilde{W}(w,a,x,y) - U\left(a,x\right)\right]^{\eta}\left[\tilde{J}\left(w,a,x,y\right) - V\left(y\right)\right]^{1-\eta}.$$

The first order condition for the bargaining problem is

$$\eta\left(\tilde{J}(w,a,x,y)-V(y)\right)\tilde{W}_w(w,a,x,y)+(1-\eta)\left(\tilde{W}(w,a,x,y)-U(a,x)\right)\tilde{J}_w(w,a,x,y)=0.$$

From the Envelop theorem, we have

$$\tilde{W}_w(w,a,x,y)=\frac{1}{1+\rho\Delta}\left\{(1-\Delta\sigma)W_a(a+(ra+w-c)\Delta,x,y)\Delta+\Delta\sigma U_a(a+(ra+w-c)\Delta,x)\Delta\right\},$$

and

$$\tilde{J}_w(w,a,x,y)=-\Delta+\frac{1}{1+\rho\Delta}(1-\Delta\sigma)J_a(a+(ra+w-\tilde{c}^e(w,a,x,y))\Delta,x,y)(1-\tilde{c}^e_w(w,a,x,y))\Delta.$$

Insert the expressions for $\tilde{W}$, $\tilde{J}$, $\tilde{W}_w$, and $\tilde{J}_w$ into the Bargaining FOC, divide both sides by Δ and take the limit $\Delta\to 0$,

$$\begin{aligned}&\eta\left\{f(x,y)-w+(ra+w-\tilde{c}^e)J_a(a,x,y)+\sigma[V(y)-J(a,x,y)]-\rho V(y)\right\}W_a(a,x,y)\\ &=(1-\eta)\left(u(\tilde{c}^e)+(ra+w-\tilde{c}^e)W_a(a,x,y)+\sigma[U(a,x)-W(a,x,y)]-\rho U(a,x)\right)\{1-J_a(a,x,y)\},\end{aligned}$$

where we have used the result that $\lim_{\Delta\to 0}\tilde{c}^e_w(w,a,x,y;\Delta)=0$. This is true because the optimal consumption policy is characterized by its first order condition

$$u'(\tilde{c}^e)=\frac{1}{1+\rho\Delta}\left\{(1-\Delta\sigma)W_a(a+(ra+w-\tilde{c}^e)\Delta,x,y)+\Delta\sigma U_a(a+(ra+w-\tilde{c}^e)\Delta,x)\right\}.$$

Notice that as $\Delta\to 0$, the limiting FOC becomes $\lim_{\Delta\to 0}u'(\tilde{c}^e)=W_a(a,x,y)$. Under mild technical conditions,

$$\lim_{\Delta\to 0}\frac{\partial\tilde{c}^e}{\partial w}(w,a,x,y;\Delta)=\frac{\partial}{\partial w}\lim_{\Delta\to 0}\tilde{c}^e(w,a,x,y;\Delta)=\frac{\partial}{\partial w}u'^{(-1)}(W_a(a,x,y))=0.$$

It is helpful to recognize that as $\Delta\to 0$,

$$\frac{\tilde{J}_w(w,a,x,y)}{\tilde{W}_w(w,a,x,y)}\to\frac{J_a(a,x,y)-1}{W_a(a,x,y)}.$$

This could be easily seen if one plugs in the expressions for $\tilde{J}_w$ and $\tilde{W}_w$ derived from the Envelop theorem. Notice that the bargaining FOC can now be written as

$$\eta[J(a,x,y)-V(y)]W_a(a,x,y)=(1-\eta)[W(a,x,y)-U(a,x)](1-J_a(a,x,y)),$$

which simplifies the bargaining FOC to

$$\begin{aligned}&\eta\left\{f\left(x,y\right)-w+\left(ra+w-\tilde{c}^{e}\right)J_{a}(a,x,y)-\rho V\left(y\right)\right\}W_{a}(a,x,y)\\=&\left(1-\eta\right)\left(u\left(\tilde{c}^{e}\right)+\left(ra+w-\tilde{c}^{e}\right)W_{a}(a,x,y)-\rho U\left(a,x\right)\right)\left(1-J_{a}(a,x,y)\right).\end{aligned}$$

The bargained wage can be solved to:

$$\begin{aligned}w=&\eta\frac{\left\{f\left(x,y\right)+\left(ra-\tilde{c}^{e}\right)J_{a}(a,x,y)-\rho V\left(y\right)\right\}}{1-J_{a}(a,x,y)}\\&-\left(1-\eta\right)\frac{u\left(\tilde{c}^{e}\right)+\left(ra-\tilde{c}^{e}\right)W_{a}(a,x,y)-\rho U\left(a,x\right)}{W_{a}}.\end{aligned}\tag{15}$$

B Algorithmic Appendix

B.1 HJB Equations

Rewrite $W\left(a,x,y\right)$ as the employed value, and $U\left(a,x\right)$ as the unemployed value. The HJB equations are $\rho W\left(w,a,x,y\right)->c_{w}$

$$\begin{aligned}\rho W\left(a,x,y\right)&=\max_{c}u\left(c\right)+\delta\left[U\left(a,x\right)-W\left(a,x,y\right)\right]+\left(ra+w\left(a,x,y\right)-c\right)W_{a}\left(a,x,y\right)\\\rho U\left(a,x\right)&=\max_{c}u\left(c\right)+p\left(\theta\right)\sum_{k}\frac{d_{v}\left(k\right)}{v}\left[W\left(a,x,y_{k}\right)-U\left(a,x\right)\right]^{+}+\left(ra+b\left(x\right)-c\right)U_{a}\left(a,x\right)\end{aligned}$$

with the first order conditions $u'\left(c\right)=W_{a}\left(a,x,y\right)$ and $u'\left(c\right)=U_{a}\left(a,x\right)$ respectively. The FD approximation to the HJB equations are

$$\rho W\left(a_{i},x_{j},y_{k}\right)=u\left(c_{i,j,k}\right)+\delta\left[U\left(a_{i},x_{j}\right)-W\left(a_{i},x_{j},y_{k}\right)\right]+\left(ra_{i}+w\left(a_{i},x_{j},y_{k}\right)-c_{i,j,k}\right)W_{a,}\left(a_{i},x_{j},y_{k}\right)\tag{16}$$

$$\rho U\left(a_{i},x_{j}\right)=u\left(c_{i,j}\right)+p\left(\theta\right)\sum_{k}\frac{d_{v}\left(k\right)}{v}\left[W\left(a_{i},x_{k},y_{k}\right)-U\left(a_{i},x_{j}\right)\right]^{+}+\left(ra_{i}+b\left(x_{j}\right)-c\right)U_{a}\left(a_{i},x_{k}\right)\tag{17}$$

B.2 Upwind Scheme

To compute the HJB equations, we need to approximate the derivatives of value functions numerically. Here we follow Achdou et al. (2020) and use the upwind scheme. The idea is to basically use the forward difference approximation whenever savings policy is positive, and backward difference whenever savings is negative.

Define the forward difference and backward difference as

$$W_{a,F}\left(a_i, x_j, y_k\right) = \frac{W\left(a_{i+1}, x_j, y_k\right) - W\left(a_i, x_j, y_k\right)}{\Delta_a}$$

$$W_{a,B}\left(a_i, x_j, y_k\right) = \frac{W\left(a_i, x_j, y_k\right) - W\left(a_{i-1}, x_j, y_k\right)}{\Delta_a}$$

$$\bar{W}_a\left(a_i, x_j, y_k\right) = u'\left(ra_i + w\left(a_i, x_j, y_k\right)\right)$$

We use the "upwind scheme". From the first order condition we can get $c = (u')^{-1} W_a(a, x, y)$. Define

$$s^W_{i,j,k,F} = ra_i + w\left(a_i, x_j, y_k\right) - \left(u'\right)^{-1}\left(W_{a,F}\left(a_i, x_j, y_k\right)\right)$$

$$s^W_{i,j,k,B} = ra_i + w\left(a_i, x_j, y_k\right) - \left(u'\right)^{-1}\left(W_{a,B}\left(a_i, x_j, y_k\right)\right)$$

and approximate the derivative as follows

$$W_a\left(a_i, x_j, y_k\right) = W_{a,B}\left(a_i, x_j, y_k\right)\mathbf{1}_{\left\{s^W_{i,j,k,B}<0\right\}} + W_{a,F}\left(a_i, x_j, y_k\right)\mathbf{1}_{\left\{s^W_{i,j,k,F}>0\right\}} + \bar{W}_a\left(a_i, x_j, y_k\right)\mathbf{1}_{\left\{s^W_{i,j,k,F}<0<s^W_{i,j,k,B}\right\}}. \tag{18}$$

Since W is concave in a, we have $s^W_{i,j,k,F} < s^W_{i,j,k,B}$, then at some point i we have $s^W_{i,j,k,F} < 0 < s^W_{i,j,k,B}$, in which case we set savings to 0. Plugging the expression (18) into the discretized HJB equation (16), then the HJB equation can be written as

$$\rho W_i^{jk} = u\left(c_i^{jk}\right) + \delta\left[U_i^j - W_i^{jk}\right] + \underbrace{\frac{W_{i+1}^{jk} - W_i^{jk}}{\Delta_a}}_{W_{a,F}} s_{i,F}^{jk,W+} + \underbrace{\frac{W_i^{jk} - W_{i-1}^{jk}}{\Delta_a}}_{W_{a,B}} s_{i,B}^{jk,W-}$$

In matrix notation

$$\rho W_i^{jk} = u\left(c_i^{jk}\right) + \delta\left[U_i^j - W_i^{jk}\right] + \frac{1}{\Delta_a}\left[\begin{array}{ccc} -s_{i,B}^{jk,W-}, & s_{i,B}^{jk,W-} - s_{i,F}^{jk,W+}, & s_{i,F}^{jk,W+} \end{array}\right]\left[\begin{array}{c} W_{i-1}^{jk} \\ W_i^{jk} \\ W_{i+1}^{jk} \end{array}\right] \tag{19}$$

Similarly define

$$\rho U_i^j = u\left(c_i^j\right) + p(\theta)\sum_k \frac{d_v(k)}{v}\left[W_i^{jk} - U_i^j\right]^+ + \frac{1}{\Delta_a}\left[\begin{array}{ccc} -s_{i,B}^{j,U-}, & s_{i,B}^{j,U-} - s_{i,F}^{j,U+}, & s_{i,F}^{j,U+} \end{array}\right]\left[\begin{array}{c} U_{i-1}^j \\ U_i^j \\ U_{i+1}^j \end{array}\right] \tag{20}$$

B.3 Implicit method

Let $\mathbf{W}$ denote the vector that stacks all value functions together. The implicit method updates the value functions in the following way:

$$\frac{1}{\Delta}\left(\mathbf{W}^{n+1}-\mathbf{W}^{n}\right)+\rho\mathbf{W}^{n+1}=\tilde{\mathbf{u}}\left(\mathbf{W}^{n}\right)+\mathbf{A}\left(\mathbf{W}^{n}\right)\mathbf{W}^{n+1}$$

which gives

$$\left(\left(\rho+\frac{1}{\Delta}\right)\mathbf{I}-\mathbf{A}\left(\mathbf{W}^{n}\right)\right)\mathbf{W}^{n+1}=\tilde{\mathbf{u}}\left(\mathbf{W}^{n}\right)+\frac{1}{\Delta}\mathbf{W}^{n}$$

$$\Rightarrow \mathbf{W}^{n+1}=\left(\left(\rho+\frac{1}{\Delta}\right)\mathbf{I}-\mathbf{A}\left(\mathbf{W}^{n}\right)\right)^{-1}\left(\tilde{\mathbf{u}}\left(\mathbf{W}^{n}\right)+\frac{1}{\Delta}\mathbf{W}^{n}\right)$$

Stack the value $\mathbf{W}$ where we first loop over assets $a_1,...,a_{Na}$, then over worker skills $x_1,..,x_{N_x}$, and then finally over firm type $y_1,...,y_{N_y}$ in the outer loop.

The matrix $\mathbf{A}\left(\mathbf{W}^{n}\right)$ has three components: one with respect to asset accumulation (the last terms of equations (19) and (20)), another with respect to job separation $\delta\left[U_i^j-W_i^{jk}\right]$, and the last one with respect to job matching $p\left(\theta\right)\sum_k\frac{d_v(k)}{v}\left[W_i^{jk}-U_i^j\right]^+$, which we denote as $\mathbf{A}_1$, $\mathbf{A}_2$ and $\mathbf{A}_3$ respectively, then $\mathbf{A}\left(\mathbf{W}^{n}\right)=\mathbf{A}_1+\mathbf{A}_2+\mathbf{A}_3$ such that

$$\mathbf{A}_1=\begin{bmatrix}\mathbf{A}_{1e} & 0\\ 0 & \mathbf{A}_{1u}\end{bmatrix}$$

$$\mathbf{A}_{1e}=\begin{bmatrix}\beta_1^{11,W} & \gamma_1^{11,W} & 0 & \cdots & 0\\ \alpha_2^{11,W} & \beta_2^{11,W} & \gamma_2^{11,W} & 0 & \cdots\\ 0 & \alpha_3^{11,W} & \beta_3^{11,W} & \gamma_3^{11,W} & 0\\ \vdots & \ddots & \ddots & \ddots & \ddots\\ 0 & \ddots & \ddots & \alpha_{Na}^{NxNy,W} & \beta_{Na}^{N_xN_y,W}\end{bmatrix},\mathbf{A}_{1u}=\begin{bmatrix}\beta_1^{1,U} & \gamma_1^{1,U} & 0 & \cdots & 0\\ \alpha_2^{1,U} & \beta_2^{1,U} & \gamma_2^{1,U} & 0 & 0\\ 0 & \alpha_3^{1,U} & \beta_3^{1,U} & \gamma_3^{1,U} & 0\\ \ddots & \ddots & \ddots & \ddots & \ddots\\ 0 & \cdots & 0 & \beta_{Na}^{N_x,U} & \gamma_{N_a}^{N_x,U}\end{bmatrix}$$

where

$$\alpha_i^{jk,W}=\frac{-s_{i,B}^{jk,W-}}{\Delta_a}$$

$$\beta_i^{jk,W}=\frac{s_{i,B}^{jk,W-}-s_{i,F}^{jk,W+}}{\Delta_a}$$

$$\gamma_i^{jk,W}=\frac{s_{i,F}^{jk,W+}}{\Delta_a}$$

and analogously for the unemployed coefficients.

$$
\mathbf{A}_2 = \begin{bmatrix}
-\delta & 0 & \cdots & 0 & & & & & & & & & 0 & \delta & 0 & \cdots & 0 \\
0 & -\delta & 0 & 0 & & & & & & & & & \vdots & 0 & \delta & 0 & 0 \\
\vdots & \ddots & \ddots & \vdots & & & & & & & & & \vdots & \vdots & \ddots & \ddots & \vdots \\
0 & 0 & \cdots & -\delta & & & & & & & & & \vdots & 0 & \cdots & 0 & \delta \\
\vdots & \ddots & \ddots & \ddots & \ddots & & & & & & & & \vdots & \delta & 0 & \cdots & 0 \\
\vdots & & & & & \ddots & & & & & & & \vdots & 0 & \delta & 0 & 0 \\
\vdots & & & & & & \ddots & & & & & & \vdots & \vdots & \ddots & \ddots & \vdots \\
\vdots & & & & & & & \ddots & & & & & \vdots & 0 & \cdots & 0 & \delta \\
\vdots & & & & & & & & & \ddots & & & \vdots & \delta & 0 & \cdots & 0 \\
\vdots & & & & & & & & & & \ddots & & \vdots & 0 & \delta & 0 & 0 \\
\vdots & & & & & & & & & & & \ddots & 0 & \vdots & \ddots & \ddots & \vdots \\
0 & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & 0 & -\delta & 0 & \cdots & 0 & \delta \\
0 & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & 0 \\
\vdots & \ddots & \ddots & \ddots & \ddots & \ddots & \ddots & \ddots & \ddots & \ddots & \ddots & \ddots & \ddots & \ddots & \ddots & \ddots & \vdots \\
0 & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & 0
\end{bmatrix}
$$

where each diagonal submatrix corresponds to a loop over asset states $a_1, ..., a_{N_a}$ and worker skills $x_1, ..., x_{N_x}$. The bottom part is a matrix of $N_1 \times N_2$ zeros where $N_1 = N_a \times N_x$ and $N_2 = N_a \times N_x \times (N_y + 1)$.

$$
\mathbf{A}_3 = p(\theta) \times \begin{bmatrix}
0 & \cdots & \cdots & \cdots & 0 \\
\vdots & \ddots & \ddots & \ddots & \vdots \\
0 & \cdots & \cdots & \cdots & 0 \\
\mathbf{A}_{31} & \mathbf{A}_{32} & \cdots & \mathbf{A}_{3N_y} & \mathbf{A}_{3N_y+1}
\end{bmatrix}
$$

where

$$
\mathbf{A}_{3k} = \begin{bmatrix}
d_v^k \mathbb{1}_1^{1k} & 0 & \cdots & \cdots & \cdots & 0 \\
0 & d_v^k \mathbb{1}_2^{1k} & \ddots & \ddots & \ddots & \vdots \\
\vdots & \ddots & \ddots & \ddots & \ddots & \vdots \\
\vdots & \ddots & \ddots & d_v^k \mathbb{1}_{N_a}^{1k} & \ddots & \vdots \\
\vdots & \ddots & \ddots & \ddots & \ddots & \vdots \\
0 & \cdots & \cdots & \cdots & 0 & d_v^k \mathbb{1}_{N_a}^{N_x k}
\end{bmatrix}
$$

$$\mathbb{1}_i^{jk} = \begin{cases} 1 & \text{if } U\left(a_i, x_j, y_k\right) > W\left(a_i, x_j\right) \\ 0 & \text{otherwise} \end{cases}$$

and

$$\mathbf{A}_{3N_y+1} = \begin{bmatrix} -\sum_k d_v^k \mathbb{1}_1^{1k} & 0 & \cdots & \cdots & \cdots & 0 \\ 0 & -\sum_k d_v^k \mathbb{1}_2^{1k} & \ddots & \ddots & \ddots & \vdots \\ \vdots & \ddots & \ddots & \ddots & \ddots & \vdots \\ \vdots & \ddots & \ddots & -\sum_k d_v^k \mathbb{1}_{N_a}^{1k} & \ddots & \vdots \\ \vdots & \ddots & \ddots & \ddots & \ddots & \vdots \\ 0 & \cdots\cdots & \cdots\cdots & \cdots\cdots & 0 & -\sum_k d_v^k \mathbb{1}_{N_a}^{N_x k} \end{bmatrix}$$

and the top part is a matrix of $N_1 \times N_2$ zeros where $N_1 = N_a \times N_x \times N_y$ and $N_2 = N_a \times N_x \times \left(N_y + 1\right)$.

Alternatively, we loop over j and k

$$\rho W_i^{jk} = u\left(c_i^{jk}\right) + \delta\left[U_i^j - W_i^{jk}\right] + \frac{1}{\Delta_a}\left[\begin{array}{ccc} -s_{i,B}^{jk,W-}, & s_{i,B}^{jk,W-} - s_{i,F}^{jk,W+}, & s_{i,F}^{jk,W+} \end{array}\right]\begin{bmatrix} W_{i-1}^{jk} \\ W_i^{jk} \\ W_{i+1}^{jk} \end{bmatrix}$$

$$\mathbf{W}_{jk}^{n+1} = \left(\left(\rho + \frac{1}{\Delta}\right)\mathbf{I} - \mathbf{A}\left(\mathbf{W}_{jk}^n\right)\right)^{-1}\left(\tilde{\mathbf{u}}\left(\mathbf{W}^n\right)_{jk}^W + \frac{1}{\Delta}\mathbf{W}_{jk}^n\right)$$

and then loop over j

$$\mathbf{U}_j^{n+1} = \left(\left(\rho + \frac{1}{\Delta}\right)\mathbf{I} - \mathbf{A}\left(\mathbf{U}_j^n\right)\right)^{-1}\left(\tilde{\mathbf{u}}\left(\mathbf{W}^n\right)_j^U + \frac{1}{\Delta}\mathbf{U}_j^n\right)$$

and then stack

$$\mathbf{W}^{n+1} = \begin{pmatrix} \mathbf{W}_{11}^{n+1} \\ \mathbf{W}_{12}^{n+1} \\ \vdots \\ \mathbf{W}_{N_x N_y}^{n+1} \\ \mathbf{U}_1^{n+1} \\ \mathbf{U}_{12}^{n+1} \\ \vdots \\ \mathbf{U}_{N_x N_y}^{n+1} \end{pmatrix}$$

This case is easy to code because $\mathbf{A}$ is standard (although $\tilde{\mathbf{u}}$ is new, but this is straightforward).

However, the loop may slowdown the algorithm.

B.4 Stationary Density

Recall the Kolmogorov Forward (KF) equations for density:

$$
\begin{aligned}
0 &= -\frac{\partial}{\partial a}\left[s_e\left(a,x,y\right)d_m\left(a,x,y\right)\right] - \delta d_m\left(a,x,y\right) + p\left(\theta\right)\frac{d_v\left(y\right)}{v}\Phi\left(a,x,y\right)d_u\left(a,x\right) \\
0 &= -\frac{\partial}{\partial a}\left[s_u\left(a,x\right)d_u\left(a,x\right)\right] - \int p\left(\theta\right)\frac{d_v\left(y\right)}{v}\Phi\left(a,x,y\right)d_u\left(a,x\right)dy + \int \delta d_m\left(a,x,y\right)dy
\end{aligned}
$$

together with the condition that density integrates to 1:

$$
1 = \int_{\underline{a}}^{\infty} d_m\left(a,x,y\right)dadxdy + \int_{\underline{a}}^{\infty} d_u\left(a,x\right)dadx
$$

as well as

$$
\begin{aligned}
d_x &= \int_{\underline{a}}^{\infty} d_m\left(a,x,y\right)dady + \int_{\underline{a}}^{\infty} d_u\left(a,x\right)da \\
d_y &= \int_{\underline{a}}^{\infty} d_m\left(a,x,y\right)dadx + d_v\left(y\right)
\end{aligned}
$$

which can be discretized as

$$
\begin{aligned}
0 &= -\frac{\partial}{\partial a}\left[s_i^{jk,W}d_i^{jk,W}\right] - \delta d_i^{jk,W} + p\left(\theta\right)\frac{d_v^k}{v}\mathbb{1}_i^{jk}d_i^{j,U} \\
0 &= -\frac{\partial}{\partial a}\left[s_i^{j,U}d_i^{j,U}\right] - p\left(\theta\right)\sum_{k=1}^{N_y}\frac{d_v^k}{v}\mathbb{1}_i^{jk}d_i^{j,U} + \delta\sum_{k=1}^{N_y}d_i^{jk,W}
\end{aligned}
$$

and

$$
\begin{aligned}
1 &= \sum_{i=1}^{N_a}\sum_{j=1}^{N_x}\sum_{k=1}^{N_y}d_i^{jk,W}\Delta_a\Delta_x\Delta_y + \sum_{i=1}^{N_a}\sum_{j=1}^{N_x}d_i^{j,U}\Delta_a\Delta_x \qquad (21) \\
d_x^j &= \sum_{i=1}^{N_a}\sum_{k=1}^{N_y}d_i^{jk,W}\Delta_a\Delta_y + \sum_{i=1}^{N_a}d_i^{j,U}\Delta_a \\
d_y^k &= \sum_{i=1}^{N_a}\sum_{j=1}^{N_x}d_i^{jk,W}\Delta_a\Delta_x + d_{vy}^k
\end{aligned}
$$

B.5 Upwind Scheme

For the derivatives, we again use the forward scheme

$$0 = -\frac{s_{i,F}^{jk,W+} d_i^{jk,W} - d_{i-1}^{jk,W} s_{i-1,F}^{jk,W+}}{\Delta_a} - \frac{d_{i+1}^{jk,W} s_{i+1,B}^{jk,W-} - d_i^{jk,W} s_{i,B}^{jk,W-}}{\Delta_a} - \delta d_i^{jk,W} + p(\theta) \frac{d_v^k}{v} \mathbb{1}_i^{jk} d_i^{j,U}$$

$$0 = -\frac{s_{i,F}^{j,U+} d_i^{j,U} - d_{i-1}^{j,U} s_{i-1,F}^{j,U+}}{\Delta_a} - \frac{d_{i+1}^{j,U} s_{i+1,B}^{j,U-} - d_i^{j,U} s_{i,B}^{j,U-}}{\Delta_a} - p(\theta) \sum_{k=1}^{N_y} \frac{d_v^k}{v} \mathbb{1}_i^{jk} d_i^{j,U} + \delta \sum_{k=1}^{N_y} d_i^{jk,W}$$

Collecting terms, we have

$$0 = d_{i-1}^{jk,W} \alpha_{i-1}^{jk,W} + d_i^{jk,W} \beta_i^{jk,W} + d_{i+1}^{jk,W} \gamma_{i+1}^{jk,W} + p(\theta) \frac{d_v^k}{v} \mathbb{1}_i^{jk} d_i^{j,U}$$

$$0 = d_{i-1}^{j,U} \alpha_{i-1}^{j,U} + d_i^{j,U} \beta_i^{j,U} + d_{i+1}^{j,U} \gamma_{i+1}^{j,U} + \delta \sum_{k=1}^{N_y} d_i^{jk,W}$$

where

$$\begin{cases} \alpha_{i-1}^{jk,W} &= \frac{s_{i-1,F}^{jk,W+}}{\Delta_a} \\ \beta_i^{jk,W} &= \frac{s_{i,B}^{jk,W-} - s_{i,F}^{jk,W+}}{\Delta_a} - \delta \\ \gamma_i^{jk,W} &= -\frac{s_{i+1}^{jk,W-}}{\Delta_a} \end{cases} \begin{cases} \alpha_{i-1}^{j,U} &= \frac{s_{i-1,F}^{j,U+}}{\Delta_a} \\ \beta_i^{j,U} &= \frac{s_{i,B}^{j,U-} - s_{i,F}^{j,U+}}{\Delta_a} - p(\theta) \sum_{k=1}^{N_y} \frac{d_v^k}{v} \mathbb{1}_i^{jk} \\ \gamma_i^{j,U} &= -\frac{s_{i+1}^{j,U-}}{\Delta_a} \end{cases}$$

Let $\mathbf{d}$ be the stacked vector of densities (arranged in the same order as $\mathbf{W}$), then the KF equations expressed using the upwind scheme can be written as

$$\mathbf{A}^T \mathbf{d} = 0 \tag{22}$$

where $\mathbf{A}^T$ is the same matrix that was defined in Section B.3.

To solve the problem of equation (22) subject to the constraints (21), we can do the following. Fix (1) either $d_i^{jk,W}$ or $d_i^{j,U}$ to be 0.1 (or any other non-zero number) for arbitrary (i, j, k); (2) then solve the system for some $\tilde{d}$ and then to renormalize

$$d_i^{jk,W} = \tilde{d}_i^{jk,W} / \left(\sum_{i=1}^{N_a} \sum_{j=1}^{N_x} \sum_{k=1}^{N_y} \tilde{d}_i^{jk,W} \Delta_a \Delta_x \Delta_y + \sum_{i=1}^{N_a} \sum_{j=1}^{N_x} \tilde{d}_i^{j,U} \Delta_a \Delta_x \right)$$

and

$$d_i^{j,U} = \tilde{d}_i^{j,U} / \left(\sum_{i=1}^{N_a} \sum_{j=1}^{N_x} \sum_{k=1}^{N_y} \tilde{d}_i^{jk,W} \Delta_a \Delta_x \Delta_y + \sum_{i=1}^{N_a} \sum_{j=1}^{N_x} \tilde{d}_i^{j,U} \Delta_a \Delta_x \right)$$

.

C Proofs of Theoretical Results

C.1 Proof of Proposition 1

Proof. From the discussion before, we know that Nash bargaining implies the following relationship for the adjusted match surplus could be written as

$$\frac{W(a,x,y) - U(a,x)}{W_a(a,x,y)} = \eta \hat{S}(a,x,y).$$

Worker optimization gives rise to the first order condition such that $W_a(a,x,y) = u'(c^e(a,x,y)) > 0$. Therefore, whether a match is formed or not, i.e., whether $\hat{S}(a,x,y) > 0$ is equivalent to whether $W(a,x,y) - U(a,x) > 0$.

Consider a such that $W(a,x,y) - U(a,x) = 0$, i.e., a marginally acceptable match. Define $\Delta(a;x,y) := W(a,x,y) - U(a,x)$. Differentiate both sides with respect to wealth a:

$$\Delta_a = W_a - U_a = u'(c^e) - u'(c^u),$$

where the arguments are suppressed for simplicity. It is obvious that for acceptable matches, $c^e > c^u$. Since the flow utility exhibits the usual concavity property $u'' < 0$, it must be that $\Delta_a = u'(c^e) - u'(c^u) < 0$.

Therefore, for any $a' > a$ we will have $\hat{S}(a',x,y) < 0$ and for any $a'' < a$ we will have $\hat{S}(a'',x,y) > 0$. □

C.2 Proof of Proposition 2

Proof. Consider $a > a'$. From Proposition 1 we know that $\Phi(a,x,y) \subset \Phi(a',x,y)$. Therefore the job finding rate of the worker of type x with wealth a is

$$\begin{aligned}
\pi_{ue}(a,x) &= p(\theta)\int \frac{d_v(y)}{v}\Phi(a,x,y)\mathrm{d}y \\
&\leq p(\theta)\int \frac{d_v(y)}{v}\Phi\left(a',x,y\right)\mathrm{d}y \\
&= \pi_{ue}\left(a',x\right)
\end{aligned}$$

□

C.3 Proof of Proposition 4

Proof. Total differentiating $W_a(a,x,y)$, we have

$$\mathrm{d}W_a(a,x,y) = W_{aa}(a,x,y)\,\mathrm{d}t$$

Apply the Envelope theorem to employed value $W(a,x,y)$ with respect to a,

$$\rho W_a(a,x,y) = \sigma\left[U_a(a,x) - W_a(a,x,y)\right] + \dot{a}W_{aa}(a,x,y) + \left[r + \omega_a(a,x,y)\right]W_a(a,x,y)\,.$$

Note that $W_a(a,x,y) = u'(c^e(a,x,y))$ and $U_a(a,x) = u'(c^u(a,x))$ by FOCs

$$u''(c^e)\,\mathrm{d}c^e = (\rho - r - \omega_a)\,u'(c^e)\,\mathrm{d}t - \sigma\left[u'(c^u) - u'(c^e)\right]\mathrm{d}t$$

Rearrange

$$\underbrace{-\frac{u''(c^e)\,c^e}{u'(c^e)}}_{\text{relative risk aversion}} \cdot \underbrace{\frac{\mathrm{d}c^e/\mathrm{d}t}{c^e}}_{\text{consumption growth}} = r - \rho + \omega_a + \sigma\left[\frac{u'(c^u)}{u'(c^e)} - 1\right]$$

Similarly, total differentiating $U_a(a,x)$, we have

$$\mathrm{d}U_a(a,x) = U_{aa}(a,x)\left[ra + b - c^u\right]\mathrm{d}t$$

Apply the Envelope theorem to unemployed value $U(a,x)$ with respect to a

$$\rho U_a(a,x) = p(\theta)\int \frac{d_v(y)}{v}\left[W_a(a,x,y) - U_a(a,x)\right]^+\mathrm{d}y + \dot{a}U_{aa}(a,x) + rU_a(a,x)$$

Plugging in FOCs

$$u''(c^u)\,\mathrm{d}c^u = (\rho - r)\,u'(c^u)\,\mathrm{d}t - p(\theta)\int_{B(a,x)}\frac{d_v(y)}{v}\left[u'(c^e) - u'(c^u)\right]\mathrm{d}y\mathrm{d}t$$

Rearrange

$$-\frac{u''(c^u)\,c}{u'(c^u)} \cdot \frac{\mathrm{d}c^u/\mathrm{d}t}{c} = r - \rho + p(\theta)\int_{B(a,x)}\frac{d_v(y)}{v}\left[\frac{u'(c^e)}{u'(c^u)} - 1\right]\mathrm{d}y$$

□

D Data Appendix

D.1 Construction of Worker and Firm Types

This section describes the method to construct multi-dimensional worker skills and job skill requirements, used by Lise and Postel-Vinay (2020).

We create 2-dimensional worker skill bundles and job skill requirement bundles using a data set combining NLSY79 job history and O*NET, following Lise and Postel-Vinay (2020).

For jobs, we

- match weekly NLSY79 job record to O*NET data which contains measures of a variety of job skill descriptors
- take the first 2 principal components of these measures in the panel
- recombine the 2 principal components so that they satisfy the following exclusion restrictions: (1) the *mathematics* measure only reflects cognitive skill requirements; (2) the *mechanical knowledge* scores only reflects manual skill requirements
- normalize the skill requirements so that each component lies in $[0, 1]$

For workers, we

- use all 10 components of individual ASVAB test scores and a measure of health (BMI)
- take the first 2 principal components of these measures
- recombine them so that (1) the ASVAB *mathematics knowledge* score only reflects cognitive skills; (2) the ASVAB *automotive and shop information* score only reflects manual skills
- normalize the skill measures so that each component lies in $[0, 1]$

In the analysis above, we only use the first component, i.e. cognitive skill/skill requirement.