Precautionary Mismatch*

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Abstract

How does wealth affect the extent to which right workers are matched with right jobs? Using NLSY79 and O*NET, we document that poorer workers are more mismatched with their jobs. We develop a framework featuring worker and firm heterogeneity, search frictions, and incomplete markets. Workers and firms face a trade-off between the worker-job fit and the speed of forming a match, and lower asset holdings make workers and firms agree upon a larger range of acceptable matches. We refer to this phenomenon as "precautionary mismatch" and show that it leads to substantial earnings and productivity dispersion between wealth-rich and wealth-poor workers of the same production type. We estimate that total output would be 3% higher in the US if all employed workers were allocated to the right jobs. In a quantitative experiment, we find that wealth transfers from incumbent workers to young labor market entrants reduce within-type earnings inequality and enhance labor productivity. Most of the productivity gains come from reduced under-matches of high-productivity workers.

Keywords: Sorting, Labor Market Mismatch, Incomplete Markets, Search and Matching
JEL Codes: J64, E21, D31

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1 Introduction

How does the labor market allocate heterogeneous workers and jobs? This question is crucial for understanding wages, productivity, and output, and has been studied by an influential literature under the assumption of risk neutrality.[1] However, transitions between employment and unemployment are arguably the largest risks facing workers, and growing empirical evidence points to the important role of wealth as the main source of self-insurance in affecting risk-averse workers' job search behavior.[2] This paper fills the gap by studying how consumption-savings decisions interact with equilibrium labor market sorting. We derive its distributional implications for wage and wealth inequality as well as aggregate implications for labor productivity.

To this end, we develop and quantify a general equilibrium random search and matching model with two-sided productive heterogeneity and incomplete markets. This framework organically integrates three classes of canonical models: an assignment model à la Becker (1973), a DMP search and matching model (Diamond, 1982; Pissarides, 1985; Mortensen and Pissarides, 1994), and an incomplete markets model in the spirit of Bewley (1986)-İmrohoroğlu (1989)-Huggett (1993)-Aiyagari (1994). However, analyzing this model has been conjectured to be intractable.[3] This paper takes the first step to advance this challenging research agenda, and makes progress both analytically through the characterization of several theoretical results (which are supported by empirical evidence) and quantitatively by building upon the computational insights from Achdou, Han, Lasry, Lions, and Moll (2022).

At the micro level, workers and firms face a trade-off between the worker-job fit and the speed of forming a match. This is because in a frictional labor market finding a perfect match takes time. Importantly, asset holdings affect such trade-off. To see this, consider a meeting where a worker has little wealth. This worker effectively has a high marginal utility of consumption and is unwilling to take risks, thus she would be happy to accept so-so matches and would not like to wait for a better match. In equilibrium, only a low wage compensation is needed to attract this worker due to her low bargaining leverage, so hiring the worker is profitable to the firm even if the worker is not really a good match to the job. Therefore, lower asset holdings

[1] Examples include Marimon and Zilibotti (1999); Shimer and Smith (2000); Gautier and Teulings (2006, 2015); Eeckhout and Kircher (2011); Lise, Meghir, and Robin (2016); Lise and Robin (2017); Hagedorn, Law, and Manovskii (2017); Bagger and Lentz (2019); Lise and Postel-Vinay (2020).

[2] See Algan et al. (2003); Rendon (2006); Card, Chetty, and Weber (2007); Chetty (2008); Basten, Fagereng, and Telle (2014); Herkenhoff (2019); Fontaine, Jensen, and Vejlin (2020); Clymo, Denderski, and Harvey (2022), among others.

[3] For example, Krusell, Mukoyama, and Şahin (2010) point out that "the computation of our model, however, is considerably more complex than for standard incomplete-markets models," even in the absence of two-sided heterogeneity and labor market sorting. More recently, Eeckhout and Sepahsalari (2018) point out that "given the difficulties analyzing sorting in the random search model (Shimer and Smith, 2000), there is little hope to address sorting on assets in random search with risk aversion." Our paper has shown hope for this line of research.

lead workers and firms to agree upon a larger range of matches that are acceptable. This is what we call *precautionary mismatch*. Moreover, such micro behavior leads to misallocation in the aggregate.[4] A poor worker takes up a job that is mismatched to him but that job could otherwise be a good match to someone else. Such misallocation leads to aggregate productivity loss. As a result, wealth inequality and labor market allocation are interacted with each other: the wealth distribution affects aggregate productivity governed by precautionary mismatch, and labor market outcomes naturally further feed back into wealth accumulation.

The mechanism at work in the model generates three key testable predictions. First, the precautionary mismatch motive implies that the matching set, i.e., the set of worker-firm pairs that are acceptable, shrinks with workers' wealth holdings. Second, wealth-poor workers have higher job finding rates as they are less picky about their job choices. Third, wealthier workers (conditional on their production type) receive higher wages due not only to higher bargaining leverage, but also to smaller mismatch with firms.[5] Using the NLSY79 and O*NET data, we find empirical evidence supporting all three predictions.

To take the model to the data, we calibrate the model to the US economy and validate the model with the empirical effects of wealth holdings on job finding rates and mismatch. Using the calibrated model, we find that precautionary mismatch leads to pronounced earnings and productivity inequality between wealth-poor and wealth-rich workers of the same production type. In particular, among the highest-skilled group, workers in the lowest wealth percentile exhibit 31.5% lower earnings and 40.8% lower output per worker than those in the top wealth percentile. This suggests that there could be substantial gains in output if mismatched workers could be efficiently reallocated. Indeed, we estimate that total output from the labor market would be 3% higher if all employed workers were allocated to the right jobs.

Finally, we conduct a policy experiment of a permanent wealth transfer from incumbent workers, who tend to be wealthy, and distribute the transfer equally to young labor market entrants, who tend to be wealth-poor. We consider a quite generous payout to each young labor market entrant of 0.5 year's worth of average earnings in the baseline economy. However, the tax burden imposed on the rest of the population to balance the budget is quite low as new entrants account for only a small fraction of the working population (e.g., the entry rate of new workers is about 1/45 of the population per year). We find that under the policy, earnings and productivity dispersion among workers of the same productivity type would decrease by over 20% in the long run, and aggregate labor productivity would increase by a modest 0.15%. Most of the

[4]The notion of "misallocation" is different from what is typically used in the macro development and firms dynamics literature, which indicates the quantity misallocation of homogeneous factor inputs (Hsieh and Klenow, 2009; Restuccia and Rogerson, 2008). This paper emphasizes the quality misallocation of heterogeneous inputs.

[5]The second (i.e. mismatch) channel is novel and enables the model to generate more frictional wage dispersion, which in turn leads to larger savings rate heterogeneity and a wealth distribution that has a much thicker tail than earnings distribution, consistent with the data.

productivity increase is attributed to the improved allocation of high-skilled workers: in the baseline economy, high-skilled young workers tend to be under-matched due to precautionary motives, and having a higher initial wealth level allows workers to conduct a more thorough job search, increasing their chances of finding better-matched jobs.

This paper makes three major contributions. The first contribution is theoretical: we provide a joint theory of wealth, wages and labor productivity. Specifically, we offer a novel perspective on the productivity effect of precautionary motives (or the lack thereof) through the lens of labor market (mis)allocation, which in turn shapes the wealth distribution as labor market transitions feed back into wealth accumulation/decumulation. By incorporating two-sided labor market heterogeneity, our model provides a well-defined notion of the "right" workers for the "right" jobs and the associated cost of labor market misallocation. In the cross-section, the model generates substantial wage dispersion due to workers' heterogeneous job search strategies and bargaining positions at different wealth levels. In doing so we also contribute to the rapidly growing research agenda on the macroeconomic implications of micro-level heterogeneity and the rich interactions between distributions and aggregates.

Our second contribution is empirical: we document the relationship between mismatch and worker wealth holdings. We construct a measure of mismatch following Lise and Postel-Vinay (2020) and Guvenen et al. (2020), and find that it is negatively associated with wealth, even after controlling for a variety of confounding factors.

Our third contribution is methodological: we develop an efficient algorithm to compute a model with rich heterogeneity, in which the equilibrium depends on an infinite-dimensional object (i.e., distributions), both in and out of the steady state. We extend the state-of-the-art continuous time method developed by Achdou et al. (2022) designed for incomplete markets models to a setting with two-sided heterogeneity, frictional sorting and endogenous wages. We derive a wage function that can be expressed by readily-computed equilibrium objects so that a guess-and-update procedure is avoided.[6] We elaborate on the relation to the existing literature in the following subsection.

Related Literature

Theoretically, our paper contributes to equilibrium theories of labor search under incomplete markets. Early foundational works including Lentz and Tranæs (2005), Rendon (2006) and Chetty (2008) study risk-averase workers' optimal savings and search decisions under unem-

[6] Krusell, Mukoyama, and Şahin (2010) points out that computation of an equilibrium where assets enter Nash bargaining problem is difficult in discrete time.

ployment risks.[7] Krusell, Mukoyama, and Şahin (2010) builds a general equilibrium model integrating incomplete-markets into a Diamond-Mortensen-Pissarides framework, which is used to evaluate a tax-financed unemployment insurance scheme. Krusell, Luo, and Ríos-Rull (2019) instead develops a directed search equilibrium model with richer risks and uses it to study employment flows over the business cycle.[8] Other recent studies including Eeckhout and Sepahsalari (2018), Griffy (2021), Ji (2021), Chaumont and Shi (2022), Baley et al. (2022) and Caratelli (2022), introduce heterogeneous productivity on either the worker side or the firm side. Compared with those previous studies, we introduce two-sided production type heterogeneity, which allows us to study the effect of wealth on sorting and thus the allocative efficiency of labor. The closest paper to ours is Herkenhoff, Phillips, and Cohen-Cole (2022), which also studies an economy with risk-averse workers and two-sided heterogeneity. Our paper is complementary to theirs in that the two papers study different ways of self-insurance and differ fundamentally in the mechanisms that generate sorting.[9] On the other hand, we also contribute to the literature on frictional sorting, including Shimer and Smith (2000), Gautier and Teulings (2006), Eeckhout and Kircher (2011) and Hagedorn, Law, and Manovskii (2017), by allowing for self-insurance to affect sorting in an incomplete-markets setting. In a nutshell, our theory nests three strands of canonical models in the macroeconomics and labor literature: an assignment model as in Becker (1973), a Diamond-Mortensen-Pissarides search and matching model, and an incomplete-markets model as in Bewley (1986), İmrohoroğlu (1989), Huggett (1993) and Aiyagari (1994).

Computationally, we develop a continuous-time technique based on Achdou et al. (2022), which applies the "Mean Field Games" theory in mathematics to cast rather complex optimization problems and equilibrium conditions in incomplete-markets models into systems of partial differential equations and a closed-form solution for wages that makes the model orders of magnitude more computationally tractable. In our model, the equilibrium depends on an infinite-dimensional distribution of worker and firms, and wages are a three-dimensional function of worker state variables. Computation of this model would be extremely costly in discrete time due to the curse of dimensionality. The continuous-time method not only enables us to compute the steady state efficiently, but also makes it possible to compute the model out of the steady state so that transitional dynamics can be studied.

Empirically, our paper is related to a large literature documenting relations between asset

[7] Lise (2013) introduces on-the-job search and finds an important asymmetry of saving behavior between the incremental wage increases generated by on-the-job search and the drop in income associated with job loss. Luo and Mongey (2019) introduce amenities in the model so that assets holdings also affect non-wage utility of accepted jobs.

[8] See Ravn and Sterk (2021) for an analysis of the HANK&SAM model that combines incomplete asset markets, nominal rigidities and search and matching frictions.

[9] We discuss the key differences in Section 4.4.3.

holdings and job search behavior (see, for example, Card, Chetty, and Weber (2007), Rendon (2006), Lentz (2009), Chetty (2008), Herkenhoff, Phillips, and Cohen-Cole (2022), among many others). These papers show overwhelmingly that increasing the ability to smooth consumption, either through unemployment insurance, wealth or access to credit, leads to longer unemployment duration and higher accepted wages. These findings provide us with an important guidance to think about the implications of the observed search behavior in the context of labor market sorting. A natural prediction from a longer unemployment duration is that the match quality of unemployed workers with new jobs also increases. To our knowledge, we are the first paper to document the effect of worker asset holdings on mismatch following an unemployment spell. Our approach to measure worker-firm mismatch follows recent papers including Lise and Postel-Vinay (2020) and Guvenen et al. (2020), which also use observable worker and job characteristics from NLSY79 and O*NET to estimate skills mismatch and effects on wages.

The rest of the paper is organized as follows. In Section 2, we describe the model and the algorithm to solve it. In Section 2.4, we discuss several key theoretical results regarding the connections between wealth, job search behavior and labor market outcomes. In Section 3, we describe the data sets we use for empirical analysis and the methods to estimate worker and firm types, and present empirical evidence on the relationship between liquid wealth, skill mismatch and wages. In Section 4 we present model calibrations and quantitative exercises. Section 5 concludes.

2 Model

This section develops and characterizes a model with the following three key elements. First, there is ex-ante heterogeneity in productive types among workers and firms, so that sorting can be studied. Second, the labor market is frictional and meetings are random, so that it takes time for workers and jobs to find "good" matches. Third, workers are risk-averse and the financial market is incomplete, so that workers need to accumulate precautionary savings in order to self-insure against unemployment risk.

2.1 Environment

Time is continuous.

Demography. The demographic structure is established as in a perpetual youth model (Blanchard, 1985) where workers exit the economy at rate δ. Exiting workers are replaced by newborns starting in unemployment with zero assets.

Preference. Workers are risk averse with an increasing, concave flow utility function $\mathfrak{u}(c)$, and maximize expected life-time utility with a discount rate ρ. Firms, on the other hand, are risk neutral and maximize the present value of expected profits discounted at the risk-free interest rate r.

Production. Upon entering the economy, workers draw the production type $x \in \mathbb{X}$ from a probability density function $d_w(x)$ and jobs draw the production type $y \in \mathbb{Y}$ from $d_j(y)$. A matched pair consisting of a worker of type x and a job of type y produces flow output $f(x, y)$. Matches are destroyed exogenously at a Poisson rate σ. At rate $\tilde{\sigma}$, the match receives the opportunity to dissolve endogenously. An unemployed worker of type x produces $b(x)$, which encompasses unemployment benefits and home production.

Search and Matching. The labor market is frictional, with search and matching occurring randomly through a *meeting* function $M(u, v)$, where u denotes the measure of unemployment and v vacancies.[10] Assuming M exhibits constant returns to scale (CRS), the meeting rate for an unemployed worker can be written as $p(\theta) := M(u, v)/u = M(1, \theta)$ and the meeting rate for a vacancy as $q(\theta) := M(u, v)/v = M(\theta^{-1}, 1) = p(\theta)/\theta$, where $\theta = v/u$ denotes labor market tightness. Wages are determined by Nash bargaining with worker bargaining power denoted by η.

Incomplete Market. Households do not have access to a full set of Arrow securities, but only to the risk-free bond with a borrowing constraint set at $\underline{a}$. There exists an annuity market where the insurance company collects the wealth of those who die and pays an annuity income flow δa to the living, which is proportional to their asset holdings. As a result, the effective return on households' assets is $r + \delta$.

2.2 Characterization

2.2.1 Distribution

Before characterizing the value functions, it is useful to define several relevant measures. For the convenience of notation, we refer to matches as m, employed workers e, unemployed workers u, producing jobs p, and vacant jobs v, using the first letter of the respective terms. For example, the density function of producing matches is denoted as $d_m(a, x, y) : \mathbb{R} \times \mathbb{X} \times \mathbb{Y} \to \mathbb{R}^+$. We define other densities in a similar fashion, with the density of employed workers as $d_e(a, x) = \int d_m(a, x, y)\,\mathrm{d}y$, the density of unemployed workers as $d_u(a, x)$, the density of producing jobs as $d_p(y) = \iint d_m(a, x, y)\,\mathrm{d}a\,\mathrm{d}x$, and the density of vacant jobs as $d_v(y)$.

[10] The distinction between *meetings* and successful *matches* is noteworthy. Once a worker and a job meet, they can decide whether or not to start production. Some meetings may not result in a successful match if the agents prefer to continue their search. The acceptance decision of forming a match is a key equilibrium object of the model.

The aggregate unemployment and vacancy rates are given by $u = \iint d_u(a,x)\,\mathrm{d}a\,\mathrm{d}x$ and $v = \int d_v(y)\,\mathrm{d}y$, respectively. Table 1 summarizes these add-up properties.

Table 1: Distribution Add-up Properties

Description	Add-up Property
Workers	$d_w(x) = \int d_u(a,x)\,\mathrm{d}a + \iint d_m(a,x,y)\,\mathrm{d}y\,\mathrm{d}a$
Unemployment	$u = \iint d_u(a,x)\,\mathrm{d}x\,\mathrm{d}a$
Jobs	$d_j(y) = d_v(y) + \iint d_m(a,x,y)\,\mathrm{d}x\,\mathrm{d}a$
Vacancies	$v = \int d_v(y)\,\mathrm{d}y$

Notes: The table summarizes the add-up properties relating $d_u(a,x)$, $d_v(y)$, u, v, and $d_m(a,x,y)$.

2.2.2 Hamilton-Jacobi-Bellman Equations

The consumption, saving, matching, and separation decisions faced by workers and firms can be characterized by a set of Hamilton-Jacobi-Bellman (henceforth HJB) equations. These form the first set of partial differential equations that characterize the equilibrium. We relegate the derivations of these value functions to Appendix I.1.

Worker Values. Employed workers make decisions on consumption-savings and whether to separate from their current job.[11] Let $U(a,x)$ denote the value of an unemployed worker of type x with wealth a, and $W(a,x,y)$ the value of an employed worker of type x with asset a working at a firm of type y. The HJB equation for an employed worker is:

$$\begin{aligned}(\rho+\delta)W(a,x,y) = \max_c \mathfrak{u}(c) &+ \sigma\left[U(a,x) - W(a,x,y)\right] + \tilde{\sigma}\left[U(a,x) - W(a,x,y)\right]^+ \\ &+ \dot{a}W_a(a,x,y) \\ \text{s.t.} \quad \dot{a} &= (r+\delta)a + \omega(a,x,y) - c \\ a &\geq \underline{a}\end{aligned} \tag{1}$$

where $[\bullet]^+ := \max\{\bullet, 0\}$. An employed worker receives flow interest and annuity $(r+\delta)a$ and wage $\omega(a,x,y)$. At Poisson rate σ, the match is destroyed for exogenous reasons and at rate $\tilde{\sigma}$, the match gets the opportunity to dissolve endogenously. The last term captures changes

[11] As we will discuss later, a worker will choose to separate if she accumulates enough assets and the job is close to the edge of the matching set, in which case the worker is better off quitting and waiting for a better match. Moreover, Nash bargaining ensures that workers' quitting decisions align with firms' separation decisions.

in the value due to changes in wealth holdings. The optimal consumption-saving decision is characterized by the first order condition

$$\mathfrak{u}'\left(c^{e}\right)=W_{a}\left(a,x,y\right). \tag{2}$$

Unemployment workers make decisions about consumption-savings and job acceptance. The HJB equation for an unemployed worker is:

$$\begin{aligned}(\rho+\delta)U\left(a,x\right)=\max_{c}\mathfrak{u}\left(c\right)+p\left(\theta\right)\int\frac{d_{v}\left(y\right)}{v}\left[W\left(a,x,y\right)-U\left(a,x\right)\right]^{+}\mathrm{d}y+\dot{a}U_{a}\left(a,x\right) \\ \text{s.t. } \dot{a}=(r+\delta)a+b(x)-c \\ a\geq\underline{a}\end{aligned} \tag{3}$$

An unemployed worker produces $b(x)$ at home and receives flow interest and annuity $(r+\delta)a$. At rate $p\left(\theta\right)$, the unemployed worker meets a vacant job randomly sampled from the vacancy distribution, and upon meeting she decides whether to match with the job by comparing the value of working $W(a,x,y)$ against the value of not working $U(a,x)$. The first order condition for the consumption-saving decision is given by

$$\mathfrak{u}'\left(c^{u}\right)=U_{a}\left(a,x\right). \tag{4}$$

Note that the matching set in unemployed workers' HJB equation is the complement of the separation set in employed workers' HJB equation. We show in Section 2.4 that the matching set shrinks as the worker becomes wealthier. When a worker is employed, she accumulates wealth and may at some point prefer to quit if the job no longer stays in the new matching set.[12]

Firm Values. Let $V\left(y\right)$ denote the value of a vacant job of type y, and $J\left(a,x,y\right)$ the value of a producing job of type y, with an employee of type x who has asset a. The HJB equation for the producing job is

$$\begin{aligned}rJ\left(a,x,y\right)=f\left(x,y\right)-\omega\left(a,x,y\right)+\left(\sigma+\delta\right)\left[V\left(y\right)-J\left(a,x,y\right)\right]+\tilde{\sigma}\left[V\left(y\right)-J\left(a,x,y\right)\right]^{+} \\ +\dot{a}^{e}\left(a,x,y\right)J_{a}\left(a,x,y\right),\end{aligned} \tag{5}$$

where $\dot{a}^{e}\left(a,x,y\right):=(r+\delta)a+\omega\left(a,x,y\right)-c^{e}\left(a,x,y\right)$ is the optimal saving policy of the employee. The firm retains the remaining output net of wage paid to the worker. The match can be

[12] The opportunity to quit is implicitly assumed in discrete time models, where decisions can only be made in each period, corresponding to $\tilde{\sigma}=1$.

separated due either to an exogenous separation shock or an endogenous optimal separation decision, in which case the job becomes vacant.[13] Note that changes in workers' wealth holdings affect firms' values as well and firms take workers' saving decisions as given.

The value of a vacant job is

$$rV(y) = q(\theta) \iint \frac{d_u(a,x)}{u} \left[J(a,x,y) - V(y)\right]^+ \mathrm{d}a\, \mathrm{d}x. \tag{6}$$

The vacancy meets an unemployed worker at rate $q(\theta)$ that is randomly drawn from the distribution of all unemployed workers.

Entrepreneurs pay a sunk investment cost of κ to set up the position. Afterwards, the job type y is realized according to a cumulative density function G. In equilibrium, the mass of jobs is determined by the following free-entry condition:

$$\kappa = \int V(y)\, \mathrm{d}G(y). \tag{7}$$

2.2.3 Wage Determination

As is standard in the search and matching literature, we assume that wages are determined by Nash bargaining. In the case of linear utility, the standard Nash bargaining solution implies a surplus function that does not depend on wage. However, this well-known proposition no longer holds in our environment as we allow for curvature in workers' utility. The associated computational challenge has been pointed out by Krusell, Mukoyama, and Şahin (2010) even in the absence of two-sided heterogeneity. We demonstrate that such intractability is circumvented in the continuous-time setup, where a generalized surplus sharing rule holds.

Proposition 1. *Nash bargaining implies the following generalized surplus sharing rule:*

$$\eta \frac{J(a,x,y) - V(y)}{1 - J_a(a,x,y)} = (1-\eta) \frac{W(a,x,y) - U(a,x)}{W_a(a,x,y)}, \tag{8}$$

where $\eta \in (0,1)$ represents the bargaining power of the worker.

The formal proof is provided in Appendix I.2. To gain intuition, it is useful to contrast our result with the common Nash solutions in the case of linear utility, where the match surplus is defined as the sum of worker's surplus and the job's surplus:

$$S(a,x,y) \coloneqq W(a,x,y) - U(a,x) + J(a,x,y) - V(y).$$

[13] Qiu (2022) documents empirical prevalence of the form of separations that vacate existing positions.

The familiar proposition of Nash bargaining is that the worker and the job are splitting the match surplus. Naturally, it only makes sense to add up the worker surplus and the firm surplus if they are measured in the same unit, as in the case of linear utility. However, in the case of concave utility, worker values are measured in present discounted utility, whereas firm values are measured in present discounted dollar value. It turns out that W_a and $1 - J_a$ are the right adjustment terms to ensure that we can add up the adjusted worker value and firm value. That is, define the adjusted surplus

$$\hat{S}(a,x,y) := \frac{1}{W_a(a,x,y)}\left[W(a,x,y) - U(a,x)\right] + \frac{1}{1 - J_a(a,x,y)}\left[J(a,x,y) - V(y)\right]. \tag{9}$$

The worker and the firm are splitting the adjusted surplus according to bargaining power η. It is obvious that W_a properly measures the marginal value of a dollar to the worker. Now we illustrate why $1 - J_a$ is the right adjustment term for the firm. Consider a marginal dollar transfer between the worker and the firm. If the worker transfers one additional dollar to the firm, there is a direct one-dollar increase in firm's value and an indirect impact to the firm through asset decumulation of the worker, i.e., $-J_a$. Thus the total marginal value of an additional dollar to the firm is properly captured by $1 - J_a$.

Equations (8) and (9) show that worker-job matching and separation decisions are privately efficient in the sense that workers' surplus from the match is positive whenever firms' surplus (and hence match surplus) is positive, and vice versa. Note that the separation decision is just the flip side of the acceptance decision: a match would end if the worker and job post separation would not have agreed to form the match.

Second, in the formal proof in Appendix I.2, we write down the Nash bargaining problem by defining values of deviating wages with tilde notations, $\tilde{W}(w,a,x,y)$. We show that

$$\frac{-\tilde{J}_w(w,a,x,y)}{\tilde{W}_w(w,a,x,y)} = \frac{1 - J_a(a,x,y)}{W_a(a,x,y)},$$

which implies that the adjusted surplus could alternatively be written as

$$\hat{S}(a,x,y) := \frac{1}{\tilde{W}_w}\left[W(a,x,y) - U(a,x)\right] + \frac{1}{-\tilde{J}_w}\left[J(a,x,y) - V(y)\right].$$

This further provides intuition to the bargaining solution—the worker's surplus is adjusted by $\tilde{W}_w$ to the dollar value, and the firm's surplus is adjusted by $-\tilde{J}_w$ to the dollar value. Workers and firms split the adjusted surplus.

Finally, notice that as the curvature of the utility function goes to 0, i.e., as the utility function goes to linear, $W_a \to 1$ and $J_a \to 0$. In this case, our adjusted surplus collapses to the standard formulation of surplus.

Appendix I.2 derives an expression for wages:

$$\omega(a,x,y) = \eta\frac{f(x,y) + ((r+\delta)a - \tilde{c}^e)J_a(a,x,y) + (\rho - r)J(a,x,y) - \rho V(y)}{1 - J_a(a,x,y)} - (1-\eta)\frac{u(\tilde{c}^e) + ((r+\delta)a - \tilde{c}^e)W_a(a,x,y) - (\rho+\delta)U(a,x)}{W_a(a,x,y)}, \tag{10}$$

which can be readily computed once we have a guess or update of the value and policy functions.

Discussion on the Assumption of Nash Bargaining

Our assumption that wages are determined through Nash bargaining follows a a substantial body of literature in the DMP tradition, including works by Krusell, Mukoyama, and Şahin (2010) and Shimer and Smith (2000), both of which are prominent models nested within our own. Employing Nash bargaining for setting wages enables us to compare our model with prior literature and assess the added value of modelling frictional sorting with incomplete markets.

There may be concerns that the Nash bargaining process requires workers' wealth to be observable to employers, leading to questions about how the results would change if we modelled the information structure as incomplete and asymmetric. However, this perfect information assumption does not materially impact our findings for two main reasons.

Firstly, the bargaining literature has indeed provided a microfoundation for Nash bargaining in the presence of heterogeneity and incomplete information. The idea is that agents can use strategic delays between offers and counteroffers to signal their types (Admati and Perry, 1987; Gul and Sonnenschein, 1988; Cramton, 1992). For instance, a high-wealth worker would decline the initial offer and defer counteroffers to signal her better outside option. This result holds when the time interval between offers is taken to be infinitesimal. Consequently, even if we were to explicitly model asymmetric information about workers' wealth, wages would still depend on wealth as reflected by Nash bargaining.

Second, we do not intend to interpret the Nash bargaining assumption in a literal sense. Instead, we use Nash bargaining as the standard device to encapsulate the idea that outside options influence wage determination. Recent studies by Caldwell and Harmon (2019) and Caldwell and Danieli (2020) provide empirical evidence that individual-level variations in outside options do indeed affect wages. Hence, we adopt the Nash bargaining wage protocol to simplify the model and avoid unnecessary complications.

2.2.4 Kolmogorov Forward Equations

We consider a stationary equilibrium wherein distributions are invariant. The steady state conditions for the distribution of workers can be characterized by two sets of Kolmogorov Forward (henceforth KF) equations. Let $\Phi(a,x,y)$ denote the matching decision of workers and firms, which equals 1 if the match is formed and 0 otherwise. Thus, $1-\Phi(a,x,y)$ represents the separation decision. The first set of KF equations characterizes the inflow and outflow for employed workers:

$$\begin{aligned} 0 = &-\frac{\partial}{\partial a}\left[\dot{a}^{e}(a,x,y)\,d_m(a,x,y)\right] - \{\sigma+\delta+\tilde{\sigma}\left[1-\Phi(a,x,y)\right]\}d_m(a,x,y) \\ &+ d_u(a,x)\,p(\theta)\,\frac{d_v(y)}{v}\Phi(a,x,y), \quad \forall\ a,x,y. \end{aligned} \tag{11}$$

The first two terms on the right hand side of equation (11) represent the outflow from the employed state (a,x,y) due to asset accumulation, exit from the economy and (both exogenous and endogenous) separation, respectively. The last term in equation (11) represents inflow due to job acceptance.

The second set of KF equations characterizes the inflow and outflow for unemployed workers:

$$\begin{aligned} 0 = &-\frac{\partial}{\partial a}\left[\dot{a}^{u}(a,x)\,d_u(a,x)\right] - p(\theta)\int\frac{d_v(y)}{v}\Phi(a,x,y)\,d_u(a,x)\,\mathrm{d}y - \delta d_u(a,x) \\ &+ \sigma\int d_m(a,x,y)\,\mathrm{d}y + \tilde{\sigma}\int\left[1-\Phi(a,x,y)\right]d_m(a,x,y)\,\mathrm{d}y + \delta d_x\cdot\mathbb{1}\{a=0\}, \quad \forall\ a,x. \end{aligned} \tag{12}$$

The first three terms on the right-hand side of equation (12) represent outflow from the unemployed state (a,x) due to asset accumulation, job finding, and exit from the economy, respectively. The next three terms represent inflow due to exogenous job separation, endogenous job separation, and newborns, respectively. We assume that agents are born with zero assets and enter the economy as unemployed. This assumption induces a realistic life-cycle pattern wherein young workers have relatively lower capacity than older workers to insure themselves against unemployment risk.

Lastly, there are add-up conditions for the density functions:

$$1 = \int_{\underline{a}}^{\infty} d_m(a,x,y)\,\mathrm{d}a\,\mathrm{d}x\,\mathrm{d}y + \int_{\underline{a}}^{\infty} d_u(a,x)\,\mathrm{d}a\,\mathrm{d}x, \tag{13}$$

as well as

$$d_w(x) = \int_{\underline{a}}^{\infty} d_m(a,x,y)\,\mathrm{d}a\,\mathrm{d}y + \int_{\underline{a}}^{\infty} d_u(a,x)\,\mathrm{d}a, \tag{14}$$

$$d_j(y) = \int_{\underline{a}}^{\infty} d_m(a,x,y)\,\mathrm{d}a\,\mathrm{d}x + d_v(y). \tag{15}$$

Namely, the sum of employed workers' (producing jobs') density and unemployed workers' (vacant jobs') density for any production type must equal the marginal density of workers' (jobs') production type.

2.3 Equilibrium

We consider a stationary sorting equilibrium. We assume an open asset market so that the economy takes the interest rate as given.[14]

2.3.1 Formal Equilibrium Definition

Given an interest rate r and marginal densities of worker and firm types d_w, d_j, a stationary sorting equilibrium consists of a set of value functions $\{W(a,x,y), U(a,x), J(a,x,y), V(y)\}$ for employed workers, unemployed workers, producing jobs, and vacant jobs, respectively; a set of policy functions including consumption policy $\{c^e(a,x,y), c^u(a,x)\}$ and matching acceptance decision conditional on meeting $\Phi(a,x,y)$; a wage policy $\omega(a,x,y)$; and an invariant distribution of employed workers $d_m(a,x,y)$ and unemployed workers $d_u(a,x)$, and market tightness θ such that:

1. The value functions and policy functions solve the worker and firm's optimization problems, characterized by the HJB equations (1, 3, 5, 6) and FOCs (2,4);
2. The Nash bargained wage satisfies the generalized surplus splitting rule (8) and the matching acceptance decision is $\Phi(a,x,y) \coloneqq \mathbb{1}\left\{\hat{S}(a,x,y) \geq 0\right\}$;
3. The stationary distributions satisfy the Kolmogorov Forward equations (11-12) and add-up conditions (13–15);
4. Market tightness adjusts so that the free entry condition (7) holds.

[14]Since our focus is on sorting between workers and firms, we impose this assumption for simplicity. In the quantitative exercise, we calibrate workers' savings in the model to liquid assets in the data. To the extent that liquid assets account for only a small fraction of the asset market, an exogenous interest rate is not unrealistic.

2.3.2 Model Outputs

The model provides a joint characterization of employment, wages, and wealth distributions through equilibrium objects from the model.

First, it characterizes standard labor market flow variables of interest. The aggregate separation rate in the economy is

$$\pi_{eu} = \sigma + \tilde{\sigma} \frac{\iiint \left[1 - \Phi(a, x, y)\right] d_m(a, x, y)\, \mathrm{d}a\, \mathrm{d}x\, \mathrm{d}y}{\iiint d_m(a, x, y)\, \mathrm{d}a\, \mathrm{d}x\, \mathrm{d}y},$$

where the two terms on the right hand side represent exogenous and endogenous separation rates, respectively. The job finding rate (which differs from job contact rate) in the economy is

$$\pi_{ue} = p(\theta) \int \frac{d_v(y)}{v} \Phi(a, x, y)\, \mathrm{d}y.$$

Second, the model also allows for a joint characterization of wage and wealth. Specifically, the joint distribution of wealth and wage (among employed workers) is characterized by

$$h(a, w) = \frac{1}{e} \iint d_m(a, x, y)\, \mathbb{1}\left\{\omega(a, x, y) = w\right\} \mathrm{d}x\, \mathrm{d}y.$$

where e is the measure of employed workers defined below.

Lastly, the model allows us to study the determinants of aggregate output and productivity. The total output and measure of employed is

$$\begin{aligned} y &= \iiint f(x, y)\, d_m(a, x, y)\, \mathrm{d}a\, \mathrm{d}x\, \mathrm{d}y, \\ e &= \iiint d_m(a, x, y)\, \mathrm{d}a\, \mathrm{d}x\, \mathrm{d}y \end{aligned}$$

and average output per employed (or labor productivity) is $\overline{y} = y/e$. From these expressions we can see that aggregate labor productivity depends on $d_m(a, x, y)$, the equilibrium sorting between workers and jobs. We use the above statistics to compare labor productivity in counterfactual economies. In Section 2.4, we show that workers' wealth holdings affect the allocation of workers through their search decisions, thereby influencing the allocative efficiency of the labor market.

2.4 Theoretical Results

2.4.1 Limiting Economies

The model provides a unified framework that incorporates sorting with two-sided heterogeneity (Becker, 1973), labor-market search and matching frictions (Diamond, 1982; Mortensen and Pissarides, 1994), and incomplete markets (Bewley, 1986; Huggett, 1993; Aiyagari, 1994). It represents a generalization that nests the frictional sorting model as in Shimer and Smith (2000) and search with precautionary savings as in Krusell, Mukoyama, and Şahin (2010).

Consider the limit where the curvature of the flow utility function approaches zero. In this case, workers become risk-neutral as the flow utility function becomes linear in consumption, that is, $\mathfrak{u}(c) = c$. The resulting limiting economy collapses to the frictional sorting model with linear utility as in Shimer and Smith (2000). Alternatively, if workers are fully insured, the problem of maximizing lifetime value becomes equivalent to the problem of maximizing lifetime income. In either case, wealth becomes irrelevant to the equilibrium allocation.

Consider the limit where both worker and firm heterogeneity are eliminated. If $\mathbb{X}$ and $\mathbb{Y}$ are singletons, the model features homogeneous workers (in terms of the production types, while preserving heterogeneity concerning wealth) and firms. In this case, there is no sorting, and the model reduces to an incomplete market model with search-and-matching frictions. This specific configuration has been explored by Krusell, Mukoyama, and Şahin (2010).

2.4.2 Wealth, Job Search and Wages

This section discusses key implications of the model regarding the interplay between wealth, job search, and wages. These insights help comprehend the mechanisms through which wealth influences labor market allocation and output, as well as how the model generates an endogenous joint distribution of wealth and wages.

Proposition 2 (**Precautionary Mismatch**). *Fix an arbitrary worker type x and job type y. Suppose a is the marginal wealth level at which the adjusted match surplus is zero. Workers with wealth exceeding this level reject the match, while those with less wealth accept it. Formally, if $\hat{S}(a, x, y) = 0$, then $\hat{S}(a', x, y) < 0$ for any $a' > a$, and $\hat{S}(a'', x, y) > 0$ for any $a'' < a$.*

Proof. See Appendix I.4. □

Proposition 2 implies that the matching set is wider for workers with lower assets. Define the matching set $\mathbb{M}(a) := \{(x, y) : \Phi(a, x, y) = 1\}$. For $a < a'$, we have $\mathbb{M}(a) \supset \mathbb{M}(a')$. It is worth noting that the proposition does not hinge on specific properties of the production function or sorting pattern, as long as workers exhibit precautionary motives for self-insurance. In Figure 1, we illustrate the matching sets (highlighted in yellow) under two specific production functions. Panel (1a) plots the matching sets when the production function is supermodular, resulting in vertical worker and firm heterogeneity, while Panel (1b) plots the matching sets when the production function is circular, resulting in horizontal worker and firm heterogeneity. The shapes of the two production functions are demonstrated in Appendix IV.1.

In Panel (1a), the equilibrium allocation exhibits positive assortative matching (PAM), with matches between workers (shown on the horizontal axis) and firms (shown on the vertical axis) occurring around the diagonal. In Panel (1b), worker and jobs are essentially placed on a unit circle with 0 and 1 being the same point. The equilibrium matches are formed when worker and job types are in close proximity on the unit circle. In both cases, workers and jobs would be perfectly matched if the matches occur along the 45-degree line (i.e. where $x = y$) as a result of assortative matching. Due to search frictions, the perfect matches take a long time to materialize, thereby leading to a range of meetings surrounding the perfect matches that are acceptable for both workers and firms. The left figures in each panel show the matching sets for workers possessing the lowest wealth in the model, while the right figures for workers whose assets equal five times their annual earnings. Workers with lower wealth have wider matching sets, regardless of the properties of the production functions. We refer to this phenomenon as "precautionary mismatch".

Proposition 3 (**Wealth and Job Finding Rate**). *The job finding rate $\pi_{ue}(a, x)$ is decreasing in wealth a.*

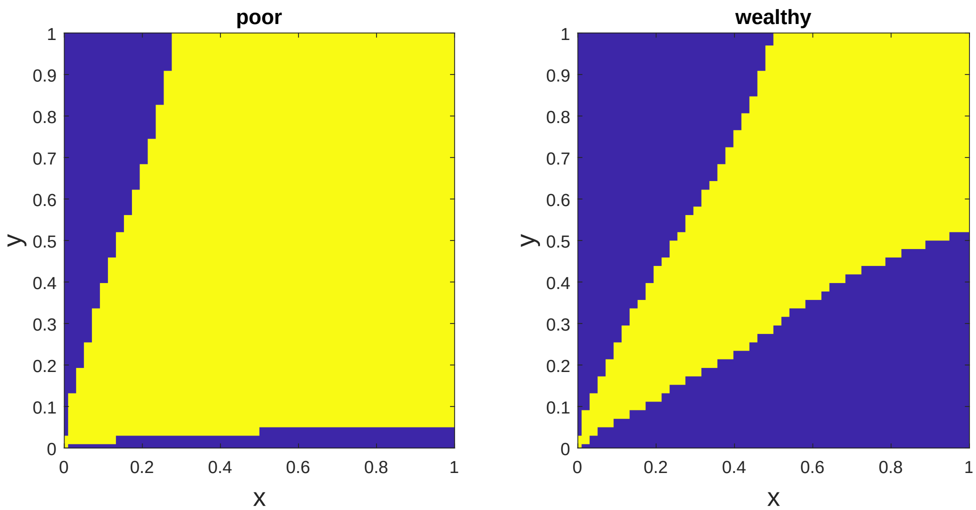
Proof. See Appendix I.5. □

Proposition 3 is a direct corollary of Proposition 2: as workers with lower wealth have wider matching sets, they are more likely to find successful matches, thereby leading to higher job finding rates. In Section 4, we test this prediction using NLSY79 data and confirm that poorer workers find jobs faster. This is consistent with empirical findings in Card, Chetty, and Weber (2007), Chetty (2008), Lise (2013), among others. The effect of wealth on job finding rate in the data thus informs the extent to which workers are willing to accept mismatch due to precautionary motives. In the calibration exercise, we replicate this moment in the model. In the aggregate, it governs the relationship between wealth holdings and the allocative efficiency in the labor market.

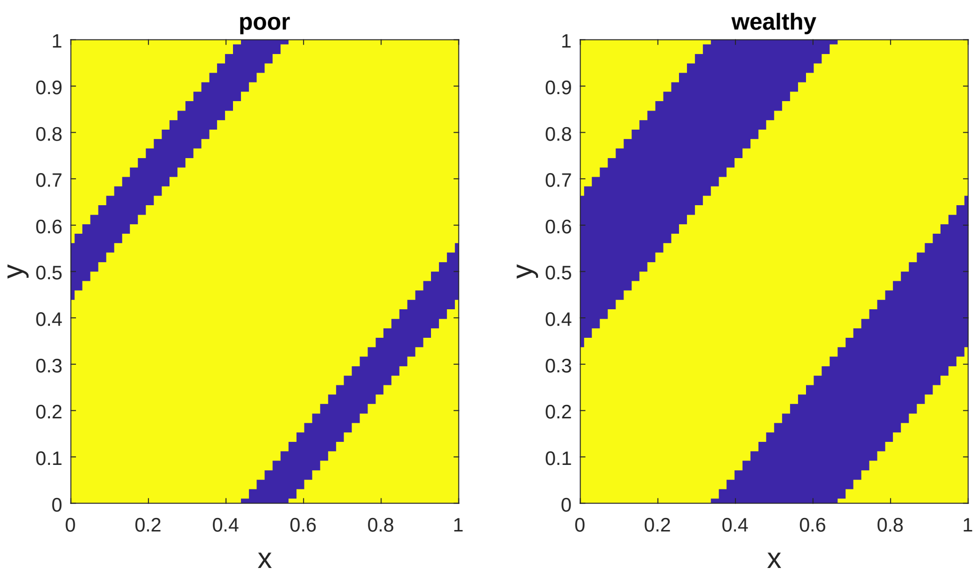
Proposition 4 (**Wealth and Wages**). *Given worker type x, the average wages of new hires, defined as $\bar{w}(a, x) = \int w(a, x, y)\, \Phi(a, x, y) d_v(y)\, \mathrm{d}y / \int \Phi(a, x, y) d_v(y)\, \mathrm{d}y$, is increasing in wealth a.*

Figure 1: Matching Sets

(a) Vertical Heterogeneity



(b) Horizontal Heterogeneity



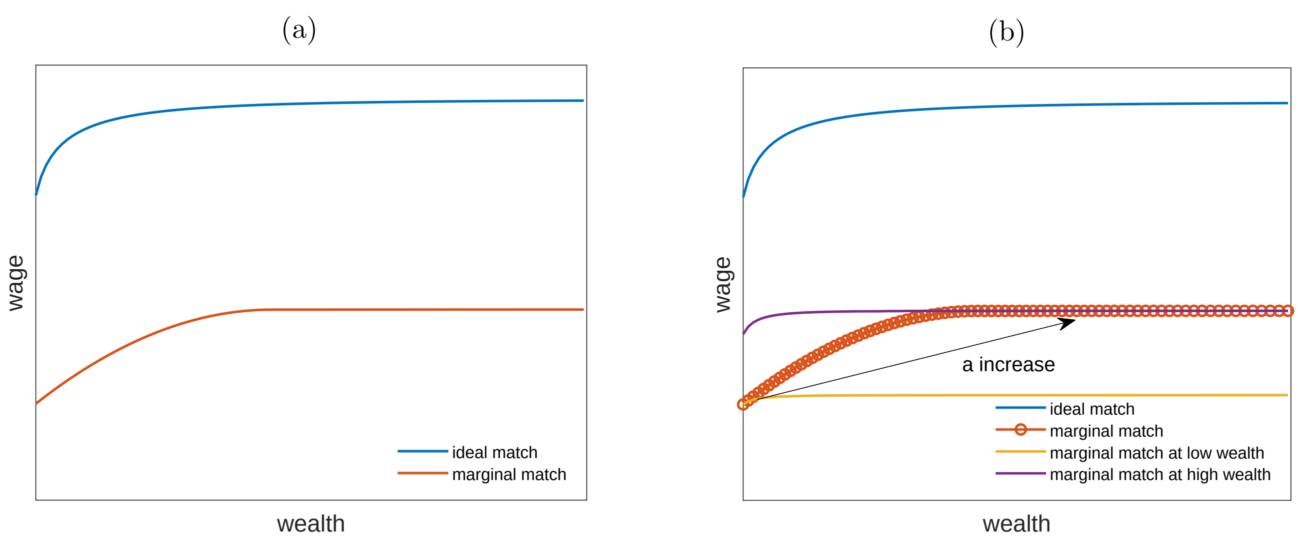
Notes: This graph shows the matching sets between workers and firms of different productivity types. Panel (1a) shows the case where agents are heterogeneous vertically and Panel (1b) shows the case where agents are heterogeneous horizontally.

We illustrate the relationship between wealth and wages of new hires in Figure 2. The blue line plots the wage function under a perfect match (where $x = y$), while the red line plots the wage function under a marginal match, which is a match at the edge of the matching set. For a marginal match, the worker and the firm are indifferent between accepting the match and staying unemployed/vacant.

The left panel of Figure 2 shows that wages for both the perfect and marginal match increases with wealth. For the perfect match, wage increases with wealth for the same reason as in Krusell, Mukoyama, and Şahin (2010): the outside option, namely the value of being unemployed, increases quickly with wealth as the precautionary motive diminishes, allowing wealthier workers to negotiate higher wages. For the marginal match, wage increases for two reasons, as shown in the right panel of Figure 2. First, for any given match (depicted by the purple or yellow line), wage increases with wealth due to bargaining. Second, as workers become wealthier, their matching set contracts, and the marginal match gets closer to the perfect match. Consequently, as wealth increases, wage increases not only along but also across different wage curves (i.e., transitioning from the yellow curve to the purple curve).

The novel mismatch channel allows for a stronger effect of wealth on wages compared to models without this channel. Prior research such as Rendon (2006) has documented that workers' reservation wages rises with wealth holdings. The empirical relationship between wealth and wages of new hires is a useful moment to validate our model calibration strategy as well as the model-implied labor market consequences of wealth holdings in Section 4.4.

Figure 2: Wealth and Wages



Notes: This graph shows wages as a function of wealth at the ideal match (blue curve), the marginal match at the corresponding wealth level (orange curve), the marginal match at the lowest wealth level (yellow curve), and the marginal match at the highest wealth level (purple curve).

Proposition 5 (**Euler Equations**). *The optimal consumption growth paths for employed and unemployed workers can be written as follows:*

$$\frac{\dot{c}^e}{c^e} = \frac{1}{\gamma}\left\{r - \rho + \omega_a + \sigma\left[\frac{u'(c^u)}{u'(c^e)} - 1\right]\right\},$$
$$\frac{\dot{c}^u}{c^u} = \frac{1}{\gamma}\left\{r - \rho - p(\theta)\int_{B(a,x)} \frac{d_v(y)}{v}\left[1 - \frac{u'(c^e)}{u'(c^u)}\right]\mathrm{d}y\right\},$$

where arguments of $\omega(a, x, y), c^u(a, x, y), c^e(a, x, y)$ are suppressed for brevity, and $B(a, x) := \{y : \Phi(a, x, y) = 1\}$ is the acceptance set of worker (a, x).

Proof. See Appendix I.6. □

These equations shed light on the factors that influence workers' saving decisions. First, a standard saving motive arises from the difference between the interest rate r and the discount rate ρ. For employed workers, additional saving motives exist due to the wealth effect on wages ω_a, and the possibility of job loss. The term enclosed in square brackets represents the precautionary savings motive, which is particularly pronounced when wealth is low or when the current wage is high. For unemployed workers, there is a dis-saving motive due to the prospect of finding a job. Notably, the dis-saving motive depends on the worker's acceptance set $B(a, x)$. All else being equal, a larger acceptance set prompts unemployed workers to dis-save more.

These propositions elucidate the mechanisms behind the endogenous joint distribution of wealth and wages generated by our model. Wealth affects wages by influencing workers' outside options and enabling job-seekers to wait for better matches, while wages in turn affect wealth by altering income flow and consumption-saving behavior. Most existing models lead to degenerate joint distributions.

3 Data and Empirical Results

Our empirical analysis utilizes a worker panel from the 1979 National Longitudinal Survey of Youth (NLSY79), a nationally representative survey conducted on individuals aged 14–22 when first interviewed in 1979. We merge the NLSY79 work history and asset information with data from the Occupational Information Network (O*NET), an occupation-level dataset containing scores on skill content for 974 occupations. This process results in a worker-occupation matched dataset with both worker and job characteristics. We construct a measure of worker skills as a proxy for worker type x, and a measure of occupational skill requirements as a proxy for job type y. The following sections detail the data sources, the construction of worker skills and job skill requirements measures, and descriptive statistics.

3.1 Data Sources and Skill Measures

O*NET

The O*NET data contains information of close to 1,000 occupations representative of the US economy, rating each occupation according to 277 different aspects, called "descriptors". These descriptors can be summarized into 9 broad categories: skills, knowledge, abilities, work activities, work context, required education levels, job interests, work styles, and work values. We keep the ratings associated with descriptors from the first 6 categories, which add up to over 200 descriptors for each occupation.

We extract job skill requirements from the O*NET descriptors following the procedure developed by Lise and Postel-Vinay (2020). First, we reduce the dimension of the occupational descriptors using Principal Component Analysis (PCA) and preserve the first three components. Next, we derive measures of cognitive, manual and interpersonal skill requirements by recombining the 3 principal components so that (1) the mathematics rating exclusively loads on cognitive skill requirements; (2) the mechanical knowledge rating exclusively loads on manual skill requirements, and (3) the social perceptiveness rating exclusively loads on interpersonal skill requirements. We take the percentile ranks of the three skill components so that each skill requirement fall within a unit-length interval $[0, 1]$. As a result, each job can be characterized by a bundle of skill requirements $\mathbf{y}$, with a higher number in each dimension indicating higher requirements of the corresponding skill.

NLSY79

We construct a monthly panel from the NLSY79 work history data. Following Farber and Gibbons (1996) and Guvenen et al. (2020), we focus on a representative cross-section of workers without military service experience, who made their initial long-term transition into the labor market during the sample period. We define a long-term transition into the labor market as the point when a worker begins working more than 1,200 hours per year for the first time. We keep observations post the last month respondents report attending school, ensuring that in the sample work decisions are not influenced by education decisions. To minimize the impact of work experience gained during education, we exclude those with more than two years of work experience prior to the completion of their schooling spell.

Similar to the approach for constructing job skill requirements, we follow Lise and Postel-Vinay (2020) and extract worker skills from a wide range of individual characteristics, including the respondents' height, weight, BMI, criminal activity, average O*NET skill requirements associated with respondents' education level, as well as test scores from the Armed Services

Vocational Aptitude Battery (ASVAB), Rosenberg self-esteem scale tests, and Rotter locus of self control scale tests. Since the three tests (ASVAB, Rosenberg and Rotter) were administered before the majority of respondents entered the labor market, the skill measure is arguably free from reverse causality wherein jobs could affect worker skills.

To construct a measure of worker skills, we run PCA on the set of individual characteristics mentioned above and retain the first three principal components. We then construct worker skills along three dimensions, namely cognitive, manual and interpersonal skills, by recombining the principal components to satisfy the following exclusion restrictions: (1) the ASVAB mathematical knowledge score only loads on cognitive skill; (2) the ASVAB automotive and shop information score only loads on manual skill, and (3) the Rosenberg self-esteem scale only loads on interpersonal skill. We then take the percentile ranks of the three dimensions, so that the worker skills on each dimension are distributed uniformly on a unit-length interval $[0, 1]$.

In addition to work history and test scores, we also gather annual history on assets from the NLSY79. Given that the NLSY79 did not begin extensive collection of wealth information until 1985, when respondents were between 20 and 28 years old, wealth is generally not observed until workers' initial entry into the labor market. We construct a measure of individuals' liquid wealth based on the sum of financial assets such as cash, deposits, mutual funds and money market accounts, and other assets more than $500, net of debts that are not asset-backed. Since asset information is not updated in each round of survey for most respondents, we linearly interpolate the amount of assets for each individual to maximize information we can use in our empirical analysis.

Appendix III.2 presents descriptive statistics for the sample.

3.2 Measuring Mismatch

In Section 2.4.2 we discussed several theoretical implications regarding wealth, job search, and wages in the model. Now we use the merged NLSY79 and O*NET data to examine the empirical relevance of these implications. First, let us provide a formal definition of the mismatch measure used for the empirical analysis.

Definition 3.1 (**Mismatch Measure**). Let $x_{i,k}$ denote the skill type of individual i in dimension k, and $y_{j,k}$ the corresponding skill requirement of job j. We define the *mismatch* between individual i and job j in dimension k as

$$m_{i,j,k} \equiv y_{j,k} - x_{i,k}, \tag{16}$$

where $m_{i,j,k} > 0$ indicates that worker i is under-qualified (or over-matched) for job j in dimension k, and vice versa. We then define the *magnitude* of mismatch between individual i and job j as

$$mm_{i,j} = \sum_k \lambda_k |m_{i,j,k}|. \tag{17}$$

where λ_k is the market weight associated with mismatch in dimension k.

In practice, we obtain λ_k's by running a Mincer wage regression on the three dimensions of mismatch. The coefficients of the three dimensions are normalized so that they sum up to 1, and λ_k corresponds to the normalized coefficient of dimension k.

Note that the definition of mismatch above concerns individual matches rather than worker allocation on an aggregate level. However, it is important to point out that aggregate labor misallocation manifests itself as individual-level mismatches, as in an economy where all workers were perfectly allocated, there would perfect sorting and thus workers types would be exactly aligned with job types so that mismatch is always 0.

3.3 Wealth and Mismatch

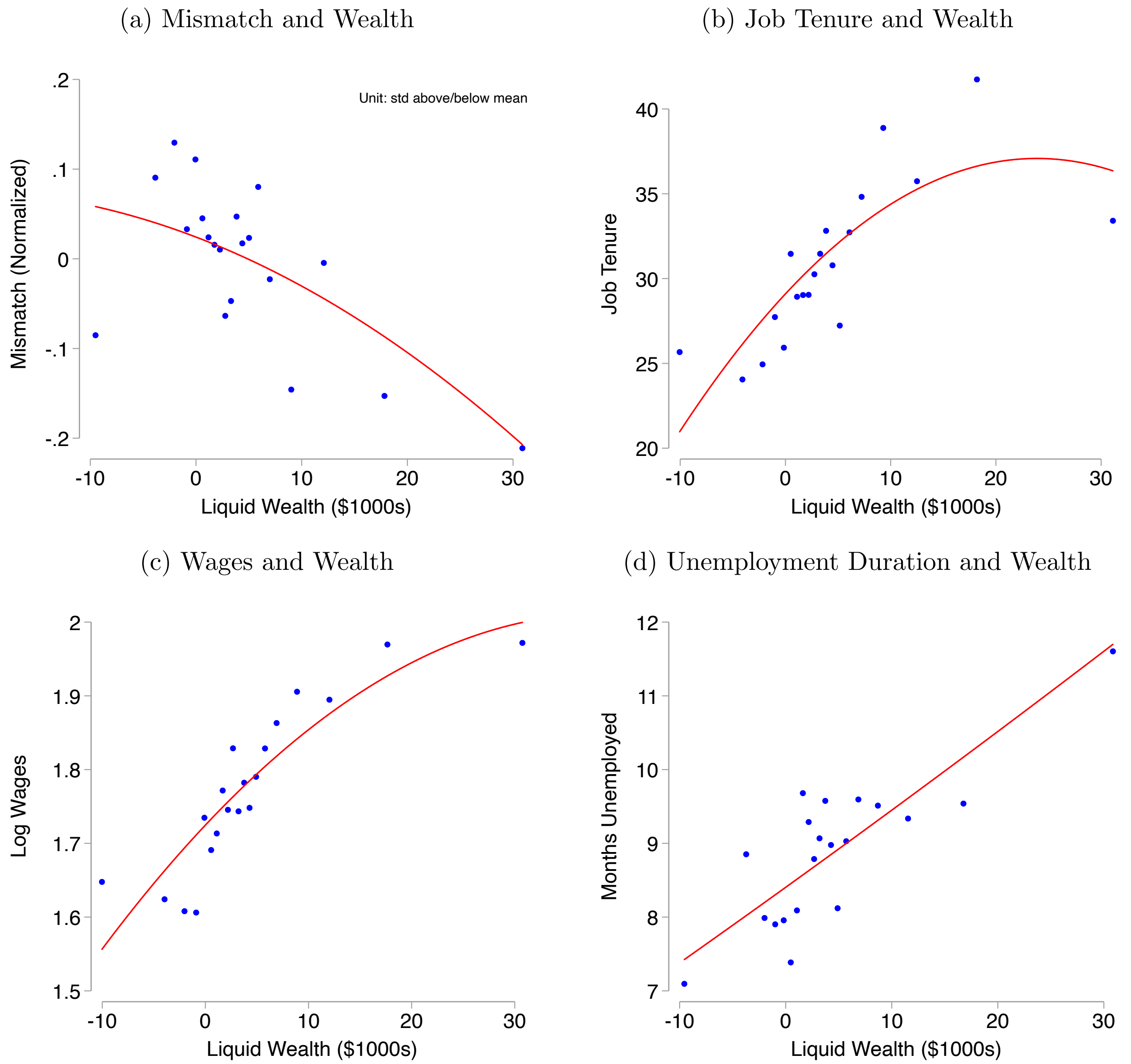
We now document the relationship between workers' liquid wealth holdings and mismatch, as outlined in Proposition 2. Figure 3a shows a binned scatter plot for those who have undergone an unemployment-to-employment (U2E) transition in the past month. The vertical axis shows the standardized mismatch measure (so that it has a mean of zero and a standard deviation of one), and the horizontal axis shows workers' liquid wealth holdings in 1982 dollars. We include a set of aggregate and individual-level controls, including quadratic functions of age, job and occupation experience, as well as race, gender, level of education, ASVAB scores, national unemployment rate, and the geographic region of the workers.

The red curve represents the best quadratic fit line to the data. On average, a \$1,000 increase in liquid wealth is associated with a 0.006-standard-deviation decrease in the magnitude of mismatch. The coefficient is significant at the 0.01 level. This finding aligns with the theoretical results from Proposition 2, which posits that wealthier workers has a smaller set of acceptable jobs, leading to lower levels of mismatch.

3.4 Wealth and Job Finding Rate

Next, we investigate the empirical relationship between wealth and the job finding rate, as discussed in Proposition 3 of Section 2.4.2. Figure 3d shows a binned scatter plot of the lengths of unemployment spells against workers' liquid wealth holdings, using the same set of controls as before.

Figure 3: Labor Market Impacts of (Liquid) Wealth



Notes: Source: NLSY79. Controls include quadratic functions of age, job experience and occupational experience, race, gender, education, ASVAB scores, national unemployment rate, region. The wage specification also controls for previous wages but the pattern is barely changed with or without this control.

Results from an OLS regression suggest that on average, a $1,000 increase in workers' liquid wealth holdings is associated with a 0.07-month, or equivalently a 2-day, increase in the duration of unemployment. The coefficient is significant at the 0.01 level. This finding aligns with Proposition 3, which posits that wealthier workers take longer to find a job. In Section 4.4, we will examine whether the calibrated model is able to closely replicate this relationship.

3.5 Wealth and Wages

We empirically test Proposition 4 from Section 2.4.2, examining whether the model-implied relationship between wealth and wages holds true in the data. Figure 3c shows a binned scatter plot of log wages upon employment against workers' liquid wealth holdings, where the controls include wages from the previous job as well as those included in the previous regressions.

Results from an OLS regression show that a $1,000 increase in liquid wealth is associated with a 1% increase in wages upon re-employment, with the coefficient being significant at the 0.01 level. We will use this statistical relationship as a validation of our model calibration strategy in Section 4.4.

3.6 Wealth and Job Tenure

Lastly, the model predicts that wealthier workers (conditional on their productivity type) tend to stay longer on their jobs upon employed. This is because the response of the marginal match with respect to asset is less elastic for wealthier workers, thus they are less likely endogenously separate from their jobs. Figure 3b shows a binned scatter plot of job tenure (in months) for each employment spell against workers' wealth holdings upon employment, controlling for the same set of variables as in the previous regressions. Note that we keep only one observation for each employment spell, and the levels of wealth holdings are taken from the first month of employment to avoid reverse causality (i.e., staying longer on a job leads to higher wealth accumulation).

As expected, higher wealth holdings are associated with longer job tenures. OLS estimates indicate that a $1,000 increase in wealth holdings is associated with a 0.3- to 0.4-month increase in job tenure.

In Appendix III.3, we report the coefficients from the above OLS regressions. We also examine the effect of wealth holdings on the extent of over- and under-match ($x < y$ and $x > y$) upon re-employment separately. Importantly, we find that higher wealth holdings have a much more significant effect on reducing under-match than over-match, which is consistent with our quantitative results in Section 4.5.1.

4 Quantitative Results

Having shown that our model's key predictions are qualitatively consistent with data, we now proceed to quantify the model.

4.1 Algorithm

The algorithm we develop to solve for the equilibrium is an extension of Achdou et al. (2022) with two-sided heterogeneity and endogenous wages for employed workers across the entire state space. Consider grids $\{a_1, a_2, \ldots, a_{N_a}\}$ for asset, $\{x_1, x_2, \ldots, x_{N_x}\}$ for worker productivity types, and $\left\{y_1, y_2, \ldots, y_{N_y}\right\}$ for firm productivity types. Suppose they are equally spaced and $\Delta_a, \Delta_x, \Delta_y$ are the step lengths.[15] The algorithm is summarized as follows.

1. Guess $d_v\left(y_k\right)$ and $d_m(a, x, y)$ (other density functions follow by add-up properties).

   We can start by guessing, for example, that $v = 1 - e = 0.1$, $d_v\left(y_k\right)/v = 1/N_y$ for all $k = 1, ..., N_y$ and $d_m(a, x, y)/e = 1/(N_a N_x N_y)$.

2. Guess the bargaining solution for each pair $\omega\left(a_i, x_j, y_k\right)$.

   We can start with $\omega\left(a, x, y\right) = \eta f\left(x, y\right)$, a fraction of the flow output.

3. Solve worker's HJB equations using the finite difference method as in Achdou et al. (2022) (see Appendix II for details).

4. Calculate the stationary distribution of workers.

   Discretize the Kolmogorov forward (KF) equation using the upwind scheme. We can then write the system of KF equations compactly in matrix form:

$$\mathbf{A}\left(\mathbf{W}^n\right)' \mathbf{d} = 0,$$

   where $\mathbf{d}$ is a stacked vector of employed workers' density and unemployed workers' density.

5. Solve firms' HJB equations also using the finite difference method (see Appendix II for details).

6. Given the value functions of workers and firms calculated from steps 3 and 5, update the wage schedule according to the expression given by equation (A1) in Appendix I.2.

7. Update the density functions based on the KF equations.

[15] The algorithm can easily accommodate uneven grids. See Appendix II for details.

8. Check whether density and wages have converged. If not, go back to Step 3.

The stationary equilibrium is solved iteratively. In each iteration, we solve for three systems of partial differential equations, two of which (workers' and firms' HJB) are non-linear and thus have to be solved iteratively, and one (workers' KF equation) is linear and can be solved without further iteration. Each system involves $N_a \times N_x \times N_y$ equations.

4.2 Parameterization

We adopt standard functional form assumptions to facilitate numerical analysis. We assume the flow utility function exhibits constant relative risk aversion (CRRA):

$$\mathfrak{u}(c) = \frac{c^{1-\gamma}}{1-\gamma} \quad , \quad \gamma > 0.$$

The aggregate meeting function is assumed to take the Cobb-Douglas form:

$$M(u, v) = \chi u^{\alpha} v^{1-\alpha}.$$

Without loss of generality, worker and job types are normalized to be uniformly distributed. To see its generality, suppose the $\tilde{F}(\tilde{x})$ and $\tilde{G}(\tilde{y})$ are the cumulative density functions of the distribution of worker and job types, respectively, with a production function $\tilde{f}(\tilde{x}, \tilde{y})$. We could redefine a type according to its rank, i.e., $x := \tilde{F}(\tilde{x})$ and $y := \tilde{G}(\tilde{y})$, and rewrite the production function accordingly $f(x, y) := \tilde{f}\left(\tilde{F}^{-1}(x), G^{-1}(y)\right)$. The distribution of the rank-based type is thus uniform, as the CDF of any random variable is uniformly distributed between 0 and 1.[16] Consistent with the empirical evidence in Hagedorn, Law, and Manovskii (2017), we specify a production function that induces positive assortative matching (PAM):

$$f(x, y) = f_0 + f_1 \left(x^{\xi} + y^{\xi}\right)^{\nu/\xi}, \quad 0 < \xi < 1, \tag{18}$$

where ξ controls the degree of complementarily between worker skills x and job skill requirements y, and ν controls the curvature of the production function across different skill levels. A smaller ξ induces stronger complementarity. A larger ν extends the right tail of the production function and thus increases wage inequality between high- and low-skilled workers.

[16] To see this, denote the transformed cumulative distribution functions as F and G such that $x \sim F$ and $y \sim G$. Consider an arbitrary $t \in [0, 1]$. We have

$$F(t) = \mathbb{P}(x \leq t) = \mathbb{P}\left(\tilde{F}(\tilde{x}) \leq t\right) = \mathbb{P}\left(\tilde{x} \leq s, \text{ for some } s \in \tilde{F}^{-1}(t)\right) = t.$$

Therefore $x \sim \mathcal{U}[0, 1]$. Similarly, $y \sim \mathcal{U}[0, 1]$.

We parameterize home production $b(x)$ to be a fraction of each worker type's lowest market output, i.e., $b(x) = b_0 \cdot f(x, \underline{y})$, where $b_0 \in [0, 1]$ and $\underline{y}$ is the minimum of $\mathbb{Y}$. This assumption ensures that no job or worker types are never matched in equilibrium.

4.3 Calibration

We set the borrowing constraint at $\underline{a} = 0$. For computation, we use grids with 200 asset grid points, 20 worker types and 20 job types. Without loss of generality, we normalize $\theta = 1$ in the stationary equilibrium, as for every θ there always exits a unique value of the vacancy posting cost that rationalizes the equilibrium. The model is calibrated to match aggregate U.S. data. Table 2 summarizes the parameter values reported at the quarterly frequency.

4.3.1 External Calibration

We set the quarterly interest rate r so that the annual rate is 2%, and set the parameter of risk aversion γ to 2. We follow Shimer (2005) and Krusell, Mukoyama, and Şahin (2010) and set both the matching function elasticity α and workers' bargaining power η to be 0.72. Note however that the Hosios condition does not guarantee efficiency in our model due to market incompleteness. The rate at which workers exit the economy δ is calibrated so that workers are expected to participate in the labor force for 45 years (age 20–65). The exogenous job separation rate is set so that the implied total (exogenous plus endogenous) job separation rate is 0.034, as in Shimer (2005). Finally, we assume the rate at which opportunities to quit voluntarily arrive is once per month on average. This assumption is made implicitly in discrete time models with monthly frequency. We have experimented with various levels of $\tilde{\sigma}$ ranging from 3 (which implies a monthly arrival frequency) to 12 (which implies a weekly arrival frequency) and have found that the model outputs of interest are insensitive to this parameter.

4.3.2 Internal Calibration

We calibrate the PAM production function specified by Equation (18) to match moments of wage dispersion in the data. We normalize the intercept f_0 to 1 without loss of generality. Since ξ affects the strength of sorting, we set its value to match the degree of frictional wage dispersion as measured by the mean-min ratio (Hornstein, Krusell, and Violante, 2011), which captures wage variations that cannot be explained by worker characteristics and hence is informative about the equilibrium matching sets. To compute this statistic in the model, we first draw a random sample of wages from the steady state distribution of employed workers, and run the following regression

$$\log(\omega_i) = \beta_0 + D_{x(i)} + \epsilon_i.$$

The left-hand side represents log wages for each worker i, and the right-hand side includes an intercept, a worker type fixed-effect, and a residual. The mean-min ratio for the residualized wage is expressed as

$$\exp\left(\overline{\epsilon_i} - \min_i(\epsilon_i)\right).$$

Next, the parameter ν, which determines the curvature of the production function, is calibrated to match the Lorenz Curve of the U.S. wage distribution, provided by Lise (2013) using SCF. Then, we jointly calibrate the scale parameter f_1 with the home production parameter b_0 so that 1) on average home production replaces 40 percent of labor market income, following Shimer (2005) and Krusell, Mukoyama, and Şahin (2010), and 2) the 90th percentile of wages is 10 times the 10th percentile of wages, as calculated from the Survey of Consumer Finances (SCF) 1989–2009.

The most pertinent form of wealth to our model is liquid wealth (liquid assets including checking, saving and money market accounts, net of liquid debts including credit card and personal loans), since illiquid wealth, such as housing, pension and businesses, cannot be readily liquidated. As such, it does not directly contribute to consumption smoothing or households' precautionary measures. Therefore, we set the discount rate ρ to match the liquid wealth to earnings ratio calculated from the SCF.

4.4 Model Validations

4.4.1 Wealth Distribution

The model endogenously yields an equilibrium distribution of liquid wealth which dictates the extent to which different types of workers are able to self-insure against unemployment risks. A natural question arises: how does this model-generated wealth distribution compare to the distribution observed in the data? In the SCF, we define liquid wealth as the sum of all liquid assets, including checking and saving accounts, directly held mutual funds, stocks and bond, net of all liquid debts, defined as credit card debts. To ensure a more consistent comparison, we narrow our focus to households with non-negative liquid wealth and those whose household heads fall within the age range of 20 to 65. This selection aligns with the model's borrowing limit of 0 and its emphasis on the working-age population.

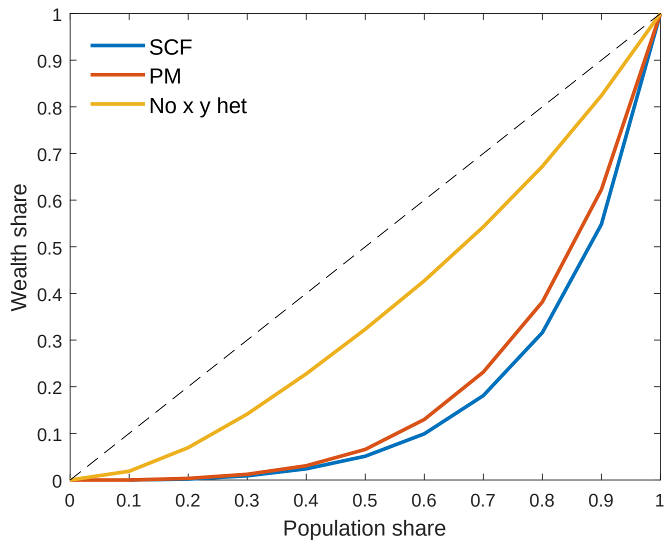
In Figure 4, we present the Lorenz curves of liquid wealth, which illustrate the distribution of liquid wealth across the population. The curves plot the shares of the population, sorted in ascending order of liquid wealth, against the fractions of liquid wealth owned by the corre-

Table 2: Calibration

Parameter	Value	Source
External Calibration		
interest rate	$r = 0.005$	annual interest rate 2%
relative risk aversion	$\gamma = 2$	common parameterization
bargaining power	$\eta = 0.72$	Shimer (2005)
matching elasticity	$\alpha = 0.72$	Shimer (2005)
dissipation rate	$\delta = 0.0056$	45-year expected working life
endogenous separation	$\tilde{\sigma} = 3$	monthly adjustment
exogenous separation	$\sigma = 0.0944$	monthly separation rate 0.034
Internal Calibration		
discount rate	$\rho = 0.012$	liquid wealth/annual earnings ratio 0.56
home production	$b_0 = 0.75$	avg $b(x)/w(x, y) = 40\%$
production function	$f_0 = 1$	normalization
production function	$f_1 = 1.78$	wage 90-10 ratio = 10
production function	$\xi = 0.8$	Hornstein, Krusell, and Violante (2011) M-m ratio
production function	$\nu = 3$	U.S. wage Lorenz Curve Lise (2013)
matching efficiency	$\chi = 4.7$	monthly job-finding rate 0.45

Notes: The table summarizes the calibrated parameters and their sources. The model is set to quarterly frequency.

Figure 4: Liquid Wealth Distribution for the Bottom 90%, Model vs. Data



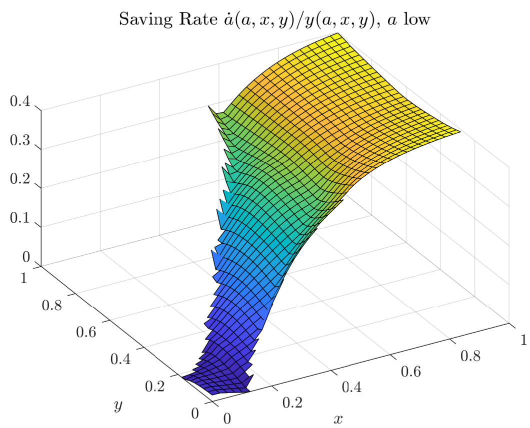
Notes: The figure shows the Lorenz curves of liquid wealth for households at the bottom 90% of the liquid wealth distribution. The Lorenz curves plot shares of the population sorted in increasing order of liquid wealth against the fraction of liquid wealth owned by that part of the population.

sponding part of the population. Specifically, the blue curve represents the distribution derived from the SCF data, the red curve represents the baseline model's wealth distribution, and the yellow curve represents an alternative version of the model where the two-sided production heterogeneity is shut down and there is a single wage for all employed workers that equals the average wage in the baseline model. We focus on households at the bottom 90% of the liquid wealth distribution.

This comparison highlights that our baseline model with two-sided heterogeneity and mismatch in a frictional labor market is capable of closely replicating the empirical liquid wealth distribution. Notably, the model generates a relatively high level of wealth concentration at the upper end while also demonstrating a corresponding lack of concentration at the lower end. By comparison, this feature is not present in a standard incomplete markets model with no production heterogeneity and much simpler income dynamics. The fact that our model replicates the empirical liquid wealth distribution offers confidence in studying the effects of policies that change the shape of wealth distribution, especially if the policies involve redistribution towards the poor as these households tend to have stronger precautionary motives and are thus more responsive to changes in wealth holdings.

The reason our model generates substantial wealth dispersion lies in the heterogeneity in the precautionary savings motive, as indicated by the Euler equations in Proposition 5. Figure 5 graphically illustrates the saving rates for different (x, y) matches and wealth levels. The saving rates are defined as $\dot{a}(a, x, y) / y(a, x, y)$, where $y(a, x, y) \equiv \omega(a, x, y) + ra$ denotes flow total income. Note that matches are only formed within matching sets, and therefore saving rates at (x, y) combinations outside the sets are not depicted.

Figure 5: Saving Rate Heterogeneity



Notes: The figure shows the saving rates of workers of different levels of productivity x employed at firms of different types y. The left panel shows the saving rates with respect to x and y when wealth is low (close to 0). The right panel shows the case when wealth is high.

Figure 5 provides two takeaways. First, at any given wealth level, there is heterogeneity in saving rates among workers of different productivity types, as well as among workers of the same productivity type but in different matches. Second, in any given match, there is also heterogeneity in saving rates for workers with different wealth holdings, which is a well-known result that poorer workers have higher saving rates as their precautionary motives are stronger. Our model provides another layer of saving rates dispersion arising from production heterogeneity, as the income differentials between wages and home productions can vary widely across workers' own production types and their jobs' types.

4.4.2 Job-Finding and Wage Elasticities in Wealth

Another key feature of the data that supports the credibility of the model is the elasticity of job finding rates and wages upon re-employment with respect to liquid wealth. These are important moments that reflect the strength of workers' precautionary motive, and how it affects their search decisions. We use monthly labor market history from NLSY79 to construct empirical moments. For job finding rates, we perform the following logit regression

$$Pr(U2E_{i,m} = 1) = F\left(\alpha_1 + \beta_1 a_{i,m} + \mathbf{X'}_{i,m}\beta_{\mathbf{1,x}} + \epsilon_{1,i,m}\right),$$

where F is the logit function, i denotes individuals, m denotes month, and $\mathbf{X}$ includes a variety of controls including a quadratic function of age, and quadratic function of work experience, race, gender, education, AFQT score, and national unemployment rate. The observations are taken during workers' unemployment spells.

For wages upon re-employment, we perform the following OLS regression

$$\log w_{i,m} = \alpha_2 + \beta_2 a_{i,m} + \mathbf{X}'_{i,m}\beta_{\mathbf{2,x}} + \epsilon_{2,i,m} \quad \text{if } U2E_{i,m} = 1$$

where observations are taken at the months when workers experience a $U2E$ transition.

In the model, we carry out corresponding regression estimates, where the controls are dummies for workers' skill types. Table (3) compares the regression estimates from the model and the NLSY79 data. The close alignment demonstrates the model's ability to replicate the effect of precautionary motives on job finding rates and re-employment wages as seen in the data.

Table 3: Elasticities w.r.t $\log(a + \sqrt{1 + a^2})$

	Model	NLSY79	Obs.
U2E: β_1	-0.193	-0.115*** (0.021)	56,786
$\log(w)$: β_2	0.090	0.096*** (0.006)	5,508

Standard errors in parentheses, * p<0.10, ** p<0.05, *** p<0.01

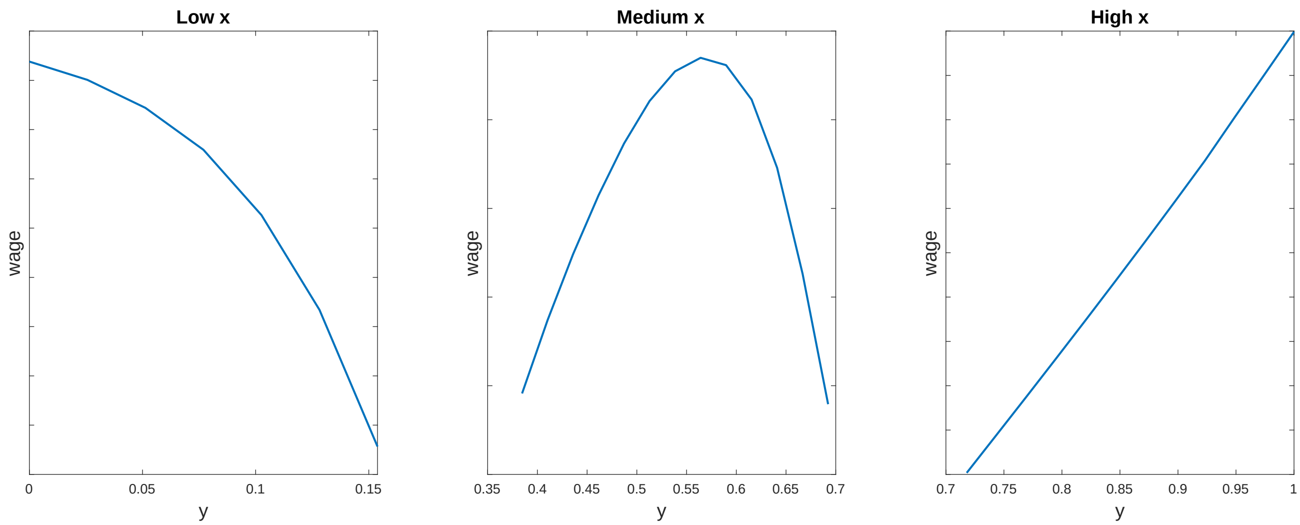
Notes: The table shows the coefficients of regressions of job finding (U2E) rates and logged wages on inverse-hyperbolic-sine-transformed wealth, controlling for worker characteristics. The second column shows the coefficients from the model, while the third column shows the coefficients from NLSY79.

4.4.3 Mismatch and Wages

Apart from the theoretical results stated in Section 2.4, another key qualitative prediction from the sorting mechanism is that wages do not always increase monotonically increasing with firm productivity. Consider, for example, a medium-skilled worker in the case of vertical heterogeneity shown in Figure 1, who are matched with firms in the middle rung of the productivity distribution but not those at the top. Due to Nash bargaining, the decision not to match with the top firms is privately efficient, as the wages these firms are willing to offer to the worker would be lower that her reservation wage. Consequently, the model predicts that for workers whose matching sets have an upper bound within the interior of $\mathcal{Y}$, wages tend to decrease as y increases towards the upper bound of the matching sets.

Figure 6 show the equilibrium wage function. Moving from left to right, the figures show wages as a function of firm productivity y for low, medium, and high productivity workers, where wealth is held constant at a given level. Wages are plotted only for firms within the matching sets of each worker type. It is evident from Figure 6 that the range of plotted wages increases with x.

Figure 6: Wages $w(a, x, y)$, a fixed



Notes: The figure shows how model-generated equilibrium wages at an arbitrary wealth level vary with firm productivity y and worker productivity x.

We can see from Figure 6 that wages tend to peak at firms that are ideal matches for each worker type. Wages are indeed lower as firms that are more mismatched with the workers, as these firms need to be compensated for the option value of waiting for better workers.

We now examine the empirical counterpart. To do so, we run a hedonic wage regression using NLSY79, specified as follows:

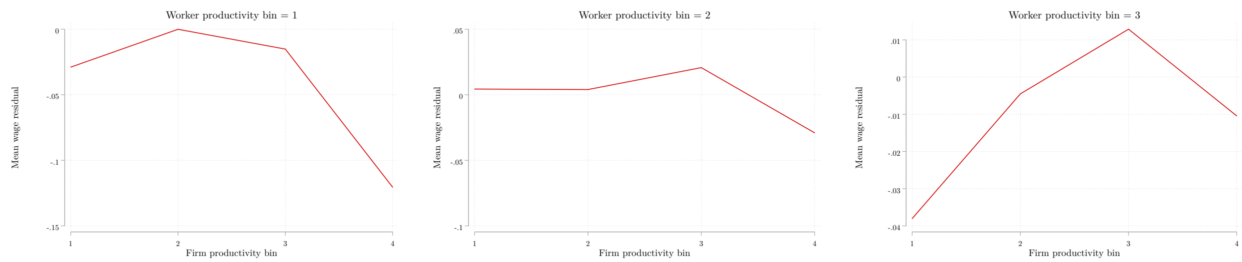
$$w_{ijt} = \alpha + \gamma_j + \mathbf{X}'_{it}\beta + \epsilon_{ijt},$$

where i denotes individual, j denotes occupation of the job, and t denotes time. We regress log wages w_{ijt} on occupation fixed effect γ_j as well as a set of worker characteristics $\mathbf{X}_{it}$ including a quadratic function of age, a quadratic function of work experience, race, gender and education dummies. Let $\tilde{w}_{ijt}$ denote the residuals from the first-stage hedonic wage regression. We then examine how $\tilde{w}_{ijt}$ varies by job skill requirements for workers of different production types. Specifically, we divide workers and jobs into 4 bins on each side, based on equal-sized cut-offs on their skill measures. We then label the bins from 1 to 4, with 1 signifying the lowest skill/skill requirement and 4 the highest.[17] Figure 7 plots the means of the wage residual $\tilde{w}_{ijt}$ by worker and job bin. We call a worker low/high-skilled if her x falls within the 1st/4th bin,

[17] Worker and job skill measures are normalized to $[0, 1]$. Workers/jobs with skill measures in $[0, 0.25)$ are assigned to bin 1, those with skill measures in $[0.25, 0.5)$ are assigned to bin 2, etc.

and medium-skilled if her x falls within the middle 2 bins.

Figure 7: Mean Wage Residuals $\tilde{w}$ by Worker and Firm Productivity



Notes: The figure shows how (residual) wages vary with worker productivity and job skill requirement. Each panel shows average (residual) wages for workers in a given productivity group employed at jobs of different skill requirement levels.

Figure 7 shows that for workers with low productivity, (residual) wages from high productivity jobs tend to be lower than those from jobs at the lower rung of the productivity distribution, while the opposite is true for workers with high productivity. This pattern aligns qualitatively with the model's predictions illustrated in Figure 6.

In addition to validating the model's prediction about sorting and wages, this result highlights the key difference to Herkenhoff, Phillips, and Cohen-Cole (2022), where wages are assumed to be a piece-rate of match-specific production, and each worker direct their search to firms of a certain productivity type. In their framework, better consumption insurance induces all workers, regardless of their productivity, to search for higher productivity firms. In our framework, the "right" jobs for each worker are not necessarily the jobs on the top rung, but those that are closer to their own levels of productivity. This is because Nash bargaining serves as a key device for generating assortative matching in equilibrium: a low-productivity worker would not opt to work at a high-productivity firm, as the firm needs to be compensated for the option value of finding a worker with higher productivity. Similarly, a low-productivity firm would choose not to hire a high-productivity worker, as the worker demands a higher wage due to the option value of finding a job with higher productivity. Thus, higher wealth holdings induce workers to find jobs closer to their own levels of productivity, as these are the "right" jobs from their own perspective. For low productivity workers, wages from high productivity firms can be lower than those offered by low productivity firms as the high productivity firms need to be compensated for the option value of waiting for better workers.

4.5 Aggregate Implications of Precautionary Mismatch

Now we are ready to address the question: how large are the effects of wealth holdings on the allocation of workers to firms, and consequently the overall productivity of the labor market?

We approach this question in two ways.

4.5.1 Wealth Holdings, Worker Allocation and Labor Market Outcomes

As shown in Proposition 2, poorer workers of any given productivity type tend to be more mismatched with their jobs. The implication about the equilibrium allocations of (wealth) poor and rich workers is, however, theoretically ambiguous, as a lack of wealth can cause mismatch in both directions (i.e., over- and under-matched).

Figure 8a shows the average matched firm type for workers in the top and bottom 1% of wealth distribution within each worker productivity type. The red solid line represents the outcomes for workers in the 99th percentile of the within-type wealth distribution, while the blue dashed line represents those in the 1st within-type wealth distribution. Overall, wealth-poor workers tend to be more under-matched than wealthy workers with the same talent levels, except for the lowest-productivity workers for whom mismatch can only take the form of over-match.[18] The gap between the wealthy and the wealth-poor arises as a result of precautionary mismatch, which is particularly pronounced for high-skilled workers.

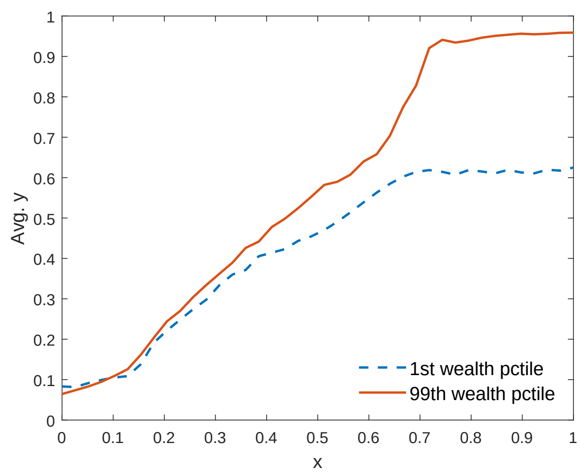
How do the differences in allocations between the wealthy and the wealth-poor translate into earnings and productivity? To answer this question, we compare the average earnings and output of workers in the top and bottom percentiles of their within-type wealth distributions. Figures 8b and 8c show average (annualized) earnings and output of wealthy and wealth-poor workers at different productivity levels. Indeed, there exists substantial earnings and productivity gap due to wealth holdings, especially for skilled workers. For the highest-skilled workers (i.e. $x = 1$), the within-group earnings gap is 31.5% and the productivity gap is even larger at 40.8%. This can be understood through the fact that for high-skilled workers, under-match not only tends to be more pronounced (as shown in Figure 8a), but is also more costly due to the curvature and supermodularity of the production function.

This comparison highlights a link between wealth and wage inequality: lower wealth induces workers to be under-matched, which in turn leads to lower earnings and productivity. At the aggregate level, it also suggests that could be achieved if mismatched workers could be reallocated to the appropriate jobs. In the following exercise, we estimate the productivity gain if all employed workers are assigned to the "right" jobs.

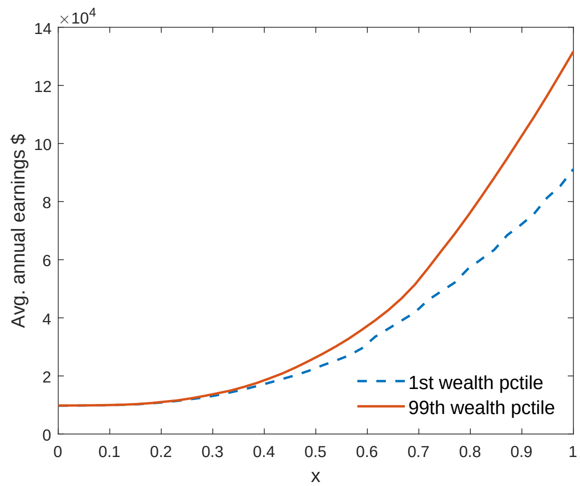
[18] Although precautionary mismatch could lead to more over-employment as suggested by Figure 1, the difference is small in practice, as the reduction in wages due to wealth-poor workers' lower bargaining position is quantitatively small. In other words, the additional incentive for firms to accept mismatched workers due to lower wage payments is weak. As a result, workers are more likely to be under-matched under the precautionary mismatch motive.

Figure 8: Effect of Wealth Holdings on Allocations, Earnings and Productivity

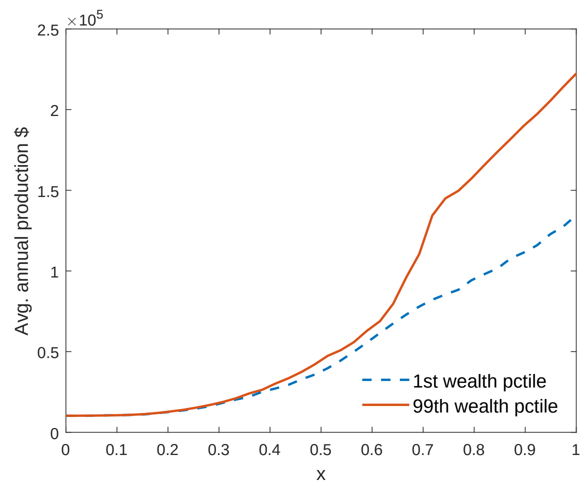
(a) Average Matched y by Wealth and x



(b) Average Earnings by Wealth and x



(c) Average Output by Wealth and x



Notes: The figure shows, among workers of each production type x, expected labor market outcomes for workers in the top and bottom 1 percent of wealth distribution. From Panel (8a) to Panel (8c), the labor market outcomes are average firm types, average (annualized) earnings and output respectively.

4.5.2 Precautionary Mismatch vs. Perfect Sorting

In this exercise, we estimate the degree of misallocation of employed workers across jobs. That is, given the set of employed workers and producing firms in equilibrium, we quantify the potential output loss compared to if all employed workers could hypothetically be assigned to the "right" jobs. This is an accounting exercise silent on the ways to achieve the efficient allocation, but it is useful to bound the effects of policies affecting consumption insurance.

To formalize this idea, denote E the total measure of employed workers and $s(x,y) := d_m(x,y)/E$ the density of matches (x,y), so that $\int s(x,y)\,dxdy = 1$. The aggregate labor productivity is thus $y = \int f(x,y)\,s(x,y)\,dxdy$. Define employed worker density $s_e(x) = \int s(x,y)\,dy$ and producing job density $s_p(y) = \int s(x,y)\,dx$, so that $\int s_e(x)\,dx = \int s_p(y)\,dy = 1$.

Definition 4.1. We define the perfect sorting allocation rule as the following:

$$\mathbb{M}(s_e(x), s_p(y)) = \max_m \int f(x,y)\,m(x,y)\,dxdy$$
$$\text{s.t.} \quad \int m(x,y)\,dy = s_e(x) \ \text{ and } \int m(x,y)\,dx = s_p(y)$$

which is in essence a linear programming problem that can be easily solved numerically.

Then the output loss due to labor misallocation can be expressed as the difference between the total outputs under the two allocation rules:

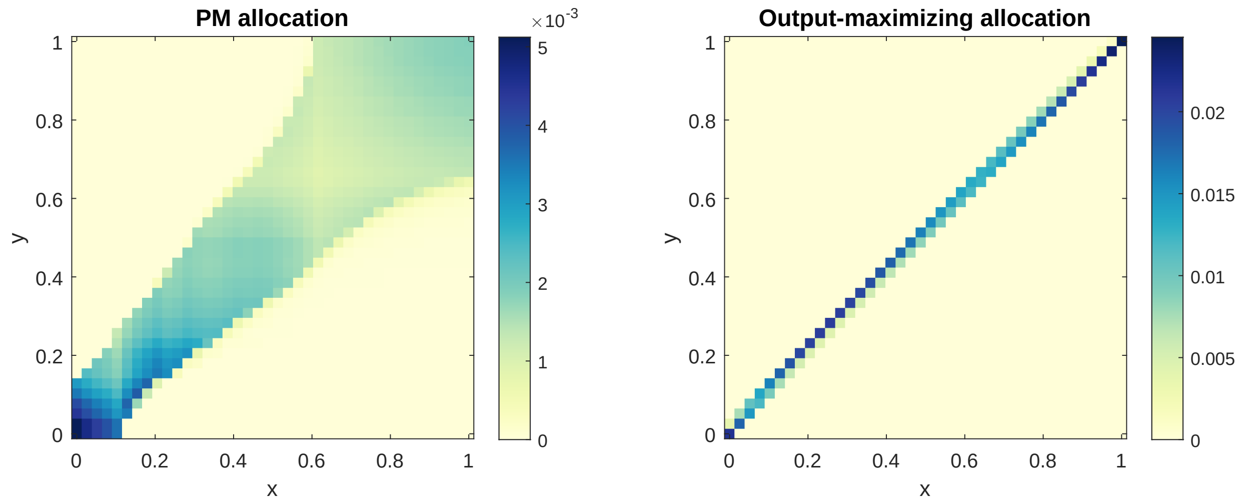
$$\int f(x,y)\left[\mathbb{M}(s_e(x), s_p(y)) - s(x,y)\right] dxdy$$

Figure 9 shows a comparison between the equilibrium labor allocation (left panel) and the output-maximizing allocation (right panel), where the x-axes represents worker skill levels and the y-axes job skill requirement levels. A lighter color indicates a higher density.

By comparing the aggregate output on the left with that on the right, we estimate that the missing output due to misallocation amounts to 2.8% of the total output in the steady state economy. Indeed, as workers and firms face search frictions, they accept a wide range of matches around the ideal ones, which, while optimal from their own perspectives, lead to mismatch and reduce the allocative efficiency of the labor market.

How are the gains in output distributed among workers by reallocating every worker to the "right" job? In Figure 10, we compare the average levels of output per worker by worker type in the steady state equilibrium (solid line) against those in the output-maximizing allocation (dashed line).

Figure 9: Equilibrium Labor Allocation vs. Output-maximizing Allocation



Notes: This figure shows the densities of matches in the steady state equilibrium (left) and in the output-maximizing allocation (right). The values of the densities are shown in the colored bars.

Perhaps surprisingly, average productivity in the steady state equilibrium is even higher for most worker types, except for the very high-skilled workers. However, remember that while inefficient from the whole labor market's standpoint, mismatch provides most workers the opportunity to be over-matched, which increases their own productivity. This comparison shows that the output cost of labor misallocation primarily stems from the under-matches of high-skilled workers, which is partially offset by the higher productivity of lower-skilled workers.

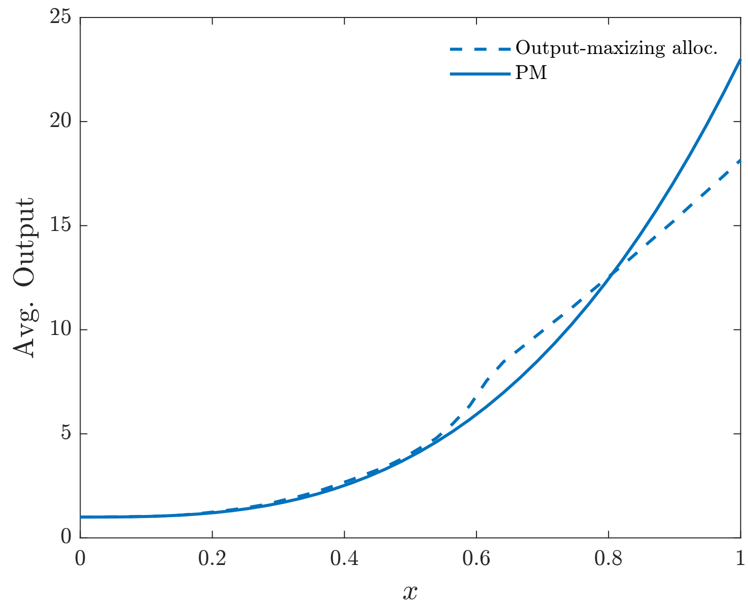
In Appendix IV.2, we provide an estimation of the extent of labor misallocation due to workers' precautionary motive by conducting an alternative accounting exercise in which all workers are assumed to follow the search strategy of the wealthiest among their own productivity types. Given that wealthy workers are well-insured against unemployment risks, they should in principle behave as if there is no precautionary motive.

4.6 Policy Experiment: Subsidy for Young Workers

Since workers enter the labor market with low wealth, they have the strongest precautionary mismatch motive. These young workers constitute a large portion of the bottom 1% of the wealth distribution, irrespective of their skills. In Section 4.5.1, we show that conditional on skill types, lower-wealth workers tend to produce less due to labor market mismatch. The productivity loss is particularly pronounced for high-skilled workers. In light of this finding, we consider an experiment that provides all young workers entering the labor market a transfer financed by a lump-sum tax imposed on the remaining population.

We assume that young workers receive an amount equal to the average liquid wealth level

Figure 10: Average Output by x: Equilibrium vs. Output-maximizing Allocation



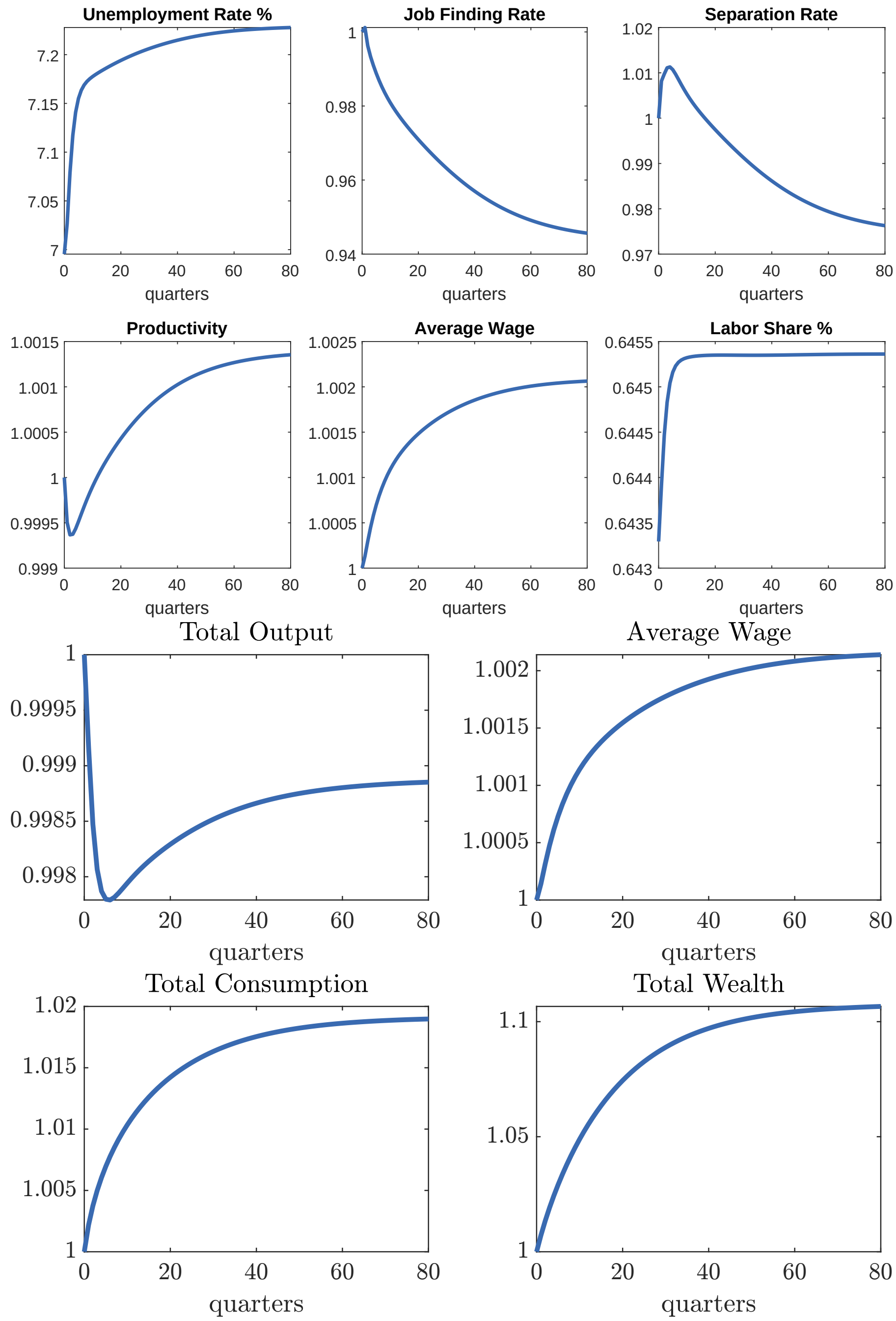
Notes: This figure shows the average output per worker for each worker type x. The solid line shows the levels according to the steady state allocation, while the dashed line shows the levels according to the output-maximizing allocation.

in the baseline economy, denoted by $\overline{a}^{PM}$, which is roughly half of the average annual earnings according to our calculation from the SCF. In the calibration, $\delta = 1/180$ of the population get replaced by newborns every quarter. Therefore, to balance the budget, the government needs to collect an amount equal to $\delta\overline{a}^{PM}$, which is roughly 0.28% of average annual earnings in the baseline equilibrium from each worker.

To gauge the time it would take to reach the new equilibrium should the government implement the policy today, as well as to envision what the new equilibrium would look like, we plot the transition path starting from the baseline equilibrium in Figure 11. The transition takes about 20 years, and the new equilibrium is characterized by a lower job finding and separation rates as young workers become pickier in choosing the right match. Productivity increases, and so does the unemployment rate. As a result, the new steady state is characterized by a 0.1% drop in (market) output. However, consumption goes up as workers receive higher wages on the job, and the fall in market output is offset by an increase in home production.

Another effect of the subsidy policy involves changes in the within-group earnings and productivity gap between the wealthy and the wealth-poor, as young workers tend to be the wealth-poorest both in the model and in the data. In Figure 12, we plot the average (annualized) earnings and output for workers in different wealth percentiles within their skill groups. The red solid line and the blue dashed line, which are the same as in Figure 8, represent the average earnings and output of workers in the top and bottom 1% of the within-group

Figure 11: Transition Path to the Equilibrium with Subsidy to the Young



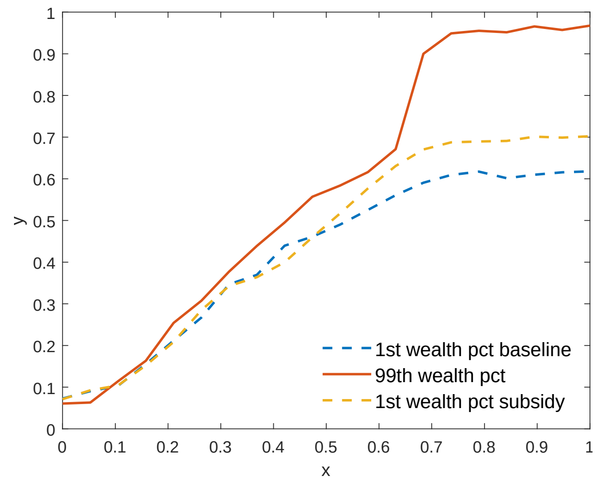
Notes: This figure shows the transition paths of economic variables from the baseline steady state to the steady state with permanent redistribution from incumbent workers to labor market entrants.

wealth distribution in the baseline economy, while the yellow dashed line shows those of the bottom 1% in the economy with subsidy. We can see that the subsidy policy workers mainly for medium- and high-skilled workers by inducing the wealth-poor within those groups to wait for higher productivity jobs. This in turn reduces the within-group earnings and output gaps for the medium-to-high-skilled workers, as they can now afford to wait longer to find higher-ranking jobs. For the highest-skilled workers ($x = 1$), earnings gap shrinks by 30.8%, while the productivity gap shrinks by 20.5%.

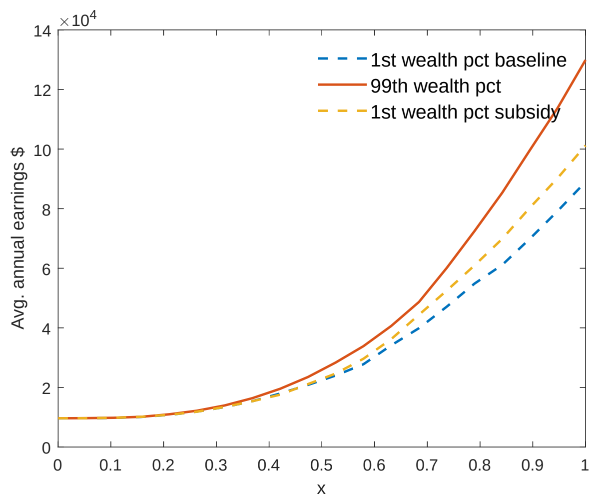
Conceptually, this policy resembles to a student loan relief program which essentially increases the capability of high-skilled (highly educated) workers to smooth consumption. However, a full comparison between the two programs is beyond the scope of this paper, as it would require modeling the interactions between workers' student loan borrowing decisions and job search decisions. Nevertheless, such an exercise would potentially help us better understand the costs and benefits of the student loan relief policy, which is currently under heated debate.

Figure 12: Labor Market Outcomes of the Wealth-Poor with vs. without Subsidy

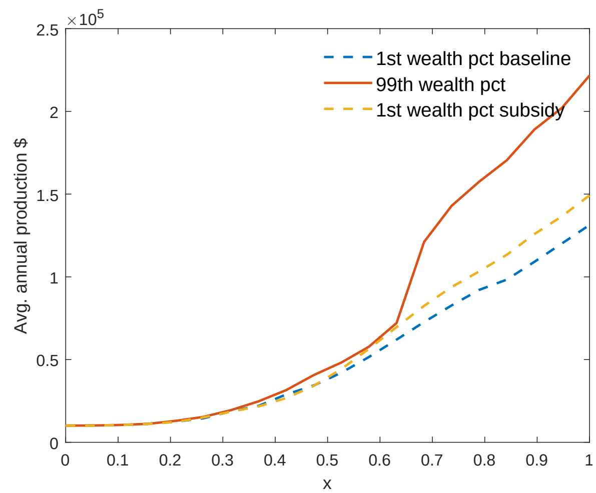
(a) Average Matched y by Wealth and x



(b) Average Earnings by Wealth and x



(c) Average Output by Wealth and x



Notes: This figure shows the labor market outcomes of workers in the top and bottom wealth percentiles in their respective productivity groups, with and without the redistribution policy. The blue dashed lines show the outcomes for the poorest workers within their productivity groups in the baseline steady state equilibrium, while the blue dashed lines show those for the poorest workers in the equilibrium with redistribution policy.

5 Conclusion

In this paper we aim to study how wealth holdings affect the allocation of workers to firms. We develop a framework with two-sided heterogeneity, search frictions and incomplete markets. Sorting occurs in equilibrium as workers and firms of different productivity types mutually agree to form matches. We find both theoretically and empirically that precautionary motives lead wealth-poor workers to speed up job search by accepting a wider range of jobs at the cost of forming worse matches, reducing allocative efficiency of the labor market. We dub this phenomenon as "precautionary mismatch."

An important takeaway from our model is that while the precautionary mismatch motive is optimal from individual workers' standpoint, it lowers the allocative efficiency of the labor market in the aggregate and reallocation of mismatched workers to their respective better-matched jobs increases overall productivity. The tension between agents' optimization decisions and labor market allocative efficiency stems from market incompleteness, and leaves open the room for policies aimed at providing better consumption insurance to impact labor market productivity.

To our knowledge, the model is the first to study frictional labor market sorting in an incomplete market. Such a framework has been known to be rather difficult to compute, let alone to estimate. We overcome this challenge by casting the model in continuous time based on the technique introduced by Achdou et al. (2022), so that the problem boils down to solving systems of linear differential equations where we can take advantage of the sparsity of the resulting matrix to speed up computation. Furthermore, using continuous time representation we can also express Nash-bargained wages using already-computed equilibrium objects, which largely reduces the complexity of equilibrium computation.

We calibrate the model to the U.S. economy and estimate that the loss of earnings and productivity due to a lack of wealth holdings can be substantial, especially for skilled workers. If we could have reallocated workers to the "right" jobs, total output would increase by about 3%. We conduct an experiment through the lens of our model in which we provide wealth transfers to young workers entering the labor market, who are the wealth-poorest both in the model and in the data. We find that such a policy would be successful at improving labor productivity by inducing young, high-skilled workers to wait for higher productivity matches, which in turn leads to higher wages and consumption. Future research may continue to explore optimal policies to provide consumption insurance and improve labor market sorting.

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I Mathematical Appendix

I.1 Derivation of HJB Equations

Consider the discrete time problem with period length of Δ

$$
\begin{aligned}
W(a,x,y) = \max_c u(c)\Delta + \frac{1}{1+\rho\Delta}\Big\{ & \underbrace{\Delta\sigma U(a',x)}_{\text{exogenous separation}} + \\
& + (1-\Delta\sigma)\max\Big[W(a',x,y), \quad \underbrace{U(a',x)}_{\text{endogenous separation}} \quad \Big]\Big\} \\
\text{s.t.} \quad a' = a + (ra + \omega(a,x,y) - c)\Delta &
\end{aligned}
$$

Consider acceptable matches $W(a,x,y) \geq U(a,x)$. As $\Delta \to 0$, $a' \to a$, continuity will preserve that $W(a',x,y) \geq U(a',x)$.

We can take the max operator off for small Δ

$$
\begin{aligned}
W(a,x,y) = u(c^e)\Delta + \frac{1}{1+\rho\Delta}\Big\{ & \Delta\sigma U(a + (ra+\omega-c^e)\Delta, x) \\
& + (1-\Delta\sigma) W(a+(ra+\omega-c^e)\Delta, x, y)\Big\}
\end{aligned}
$$

Multiply both sides by $(1+\rho\Delta)$, subtract W, and then divide them by Δ

$$
\begin{aligned}
\rho W(a,x,y) = u(c^e)(1+\rho\Delta) + & \frac{1}{\Delta}\left[W(a+(ra+\omega-c^e)\Delta,x,y) - W(a,x,y)\right] \\
& + \sigma\left[U(a+(ra+\omega-c^e)\Delta,x) - W(a+(ra+\omega-c^e)\Delta,x,y)\right]
\end{aligned}
$$

Take the limit $\Delta \to 0$,

$$
\rho W(a,x,y) = u(c^e) + \underbrace{(ra+\omega-c^e)}_{\dot{a}} W_a(a,x,y) + \sigma\left[U(a,x) - W(a,x,y)\right]
$$

Other value functions can be derived similarly.

I.2 Nash Bargaining

To derive the wage setting, we start with the discrete time problem with period length of Δ. The value for employed worker of type x with asset a that works at a job of type y for an arbitrarily deviating flow wage w (recognizing that in the following period the wage will go back to the equilibrium bargained wage) satisfies

$$\tilde{W}(w,a,x,y) = \max_c u(c)\Delta + \frac{1}{1+\rho\Delta}\{(1-\Delta\sigma)(1-\Delta\delta)W(a',x,y) + \Delta\sigma U(a',x) + \Delta\delta\cdot 0\}$$
$$\text{s.t.} \quad a' = a + (\tilde{r}a + w - c)\Delta,$$

where $\tilde{r} := r+\delta$ is the effective return. Denote the optimal consumption policy by $\tilde{c}^e(w,a,x,y)$. The Envelop condition delivers

$$\begin{aligned}\tilde{W}_w(w,a,x,y) =& \frac{1}{1+\rho\Delta}\{(1-\Delta\sigma)(1-\Delta\delta)W_a(a+(\tilde{r}a+w-\tilde{c}^e)\Delta,x,y)\Delta \\ &+ \Delta\sigma U_a(a+(\tilde{r}a+w-\tilde{c}^e)\Delta,x)\Delta\}.\end{aligned}$$

Similarly, the value for such a producing job is

$$\tilde{J}(w,a,x,y) = f(x,y)\Delta - w\Delta + \frac{1}{1+r\Delta}\left[(1-\Delta(\sigma+\delta))J(a',x,y) + \Delta(\sigma+\delta)V(y)\right],$$

where $a' = a + (\tilde{r}a + w - \tilde{c}^e(w,a,x,y))\Delta$ is taken as given from the firm's point of view. From the Envelop theorem, we have

$$\begin{aligned}\tilde{J}_w(w,a,x,y) =& -\Delta + \frac{1}{1+r\Delta}\left[1-\Delta(\sigma+\delta)\right]J_a(a+(\tilde{r}a+w-\tilde{c}^e(w,a,x,y))\Delta,x,y) \\ &\left[1-\tilde{c}^e_w(w,a,x,y)\right]\Delta.\end{aligned}$$

Under Nash bargaining, the wage policy is determined by

$$\omega(a,x,y) = \arg\max_w \left[\tilde{W}(w,a,x,y) - U(a,x)\right]^\eta \left[\tilde{J}(w,a,x,y) - V(y)\right]^{1-\eta}.$$

The first order condition for the bargaining problem is

$$\eta\left[\tilde{J}(w,a,x,y) - V(y)\right]\tilde{W}_w(w,a,x,y) + (1-\eta)\left[\tilde{W}(w,a,x,y) - U(a,x)\right]\tilde{J}_w(w,a,x,y) = 0.$$

It is helpful to recognize that as $\Delta \to 0$,

$$\frac{\tilde{J}_w(w,a,x,y)}{\tilde{W}_w(w,a,x,y)} \to \frac{J_a(a,x,y)-1}{W_a(a,x,y)}.$$

This could be easily seen if one plugs in the expressions for $\tilde{J}_w$ and $\tilde{W}_w$ derived from the Envelop theorem.

Lemma I.1.

$$\lim_{\Delta\to 0}\frac{\tilde{J}_w(w,a,x,y;\Delta)}{\tilde{W}_w(w,a,x,y;\Delta)}=\frac{J_a(a,x,y)}{W_a(a,x,y)-1}.$$

Proof. Plug in the expressions for $\tilde{J}_w$ and $\tilde{W}_w$ derived before,

$$\begin{aligned}
\frac{-\tilde{J}_w}{\tilde{W}_w} &= \frac{\Delta-\frac{1}{1+r\Delta}(1-\Delta\sigma)J_a(a+(ra+w-\tilde{c}^e(w,a,x,y))\Delta,x,y)(1-\tilde{c}^e_w(w,a,x,y))\Delta}{\frac{1}{1+\rho\Delta}\{(1-\Delta\sigma)W_a(a+(ra+w-c)\Delta,x,y)\Delta+\Delta\sigma U_a(a+(ra+w-c)\Delta,x)\Delta\}}\\
&= \frac{(1+r\Delta)-(1-\Delta\sigma)J_a(a+(ra+w-\tilde{c}^e(w,a,x,y))\Delta,x,y)}{\{(1-\Delta\sigma)W_a(a+(ra+w-c)\Delta,x,y)+\Delta\sigma U_a(a+(ra+w-c)\Delta,x)\}}\times\frac{1+\rho\Delta}{1+r\Delta}\\
&\to \frac{1-J_a(a,x,y)}{W_a(a,x,y)}
\end{aligned}$$

as $\Delta\to 0$, where $\lim_{\Delta\to 0}\tilde{c}^e_w(w,a,x,y;\Delta)=0$ is proved in Proposition 6. □

Now we can rewrite the Nash solution

$$\eta\frac{J(a,x,y)-V(y)}{1-J_a(a,x,y)}=(1-\eta)\frac{W(a,x,y)-U(a,x)}{W_a(a,x,y)}$$

as

$$\eta\frac{rJ(a,x,y)+(\rho-r)J(a,x,y)-\rho V(y)}{1-J_a(a,x,y)}=(1-\eta)\frac{\rho W(a,x,y)-\rho U(a,x)}{W_a(a,x,y)}$$

Plug in the HJB equation of rJ and ρW:

$$\begin{aligned}
&\eta\frac{f(x,y)-\omega(a,x,y)+(\tilde{r}a+\omega-\tilde{c}^e)J_a(a,x,y)+(\rho-r)J(a,x,y)-\rho V(y)}{1-J_a(a,x,y)}\\
&\qquad=(1-\eta)\frac{u(c)+(\tilde{r}a+\omega-\tilde{c}^e)W_a(a,x,y)-(\rho+\delta)U(a,x)}{W_a(a,x,y)}
\end{aligned}$$

Collecting terms, we obtain the following wage equation

$$\begin{aligned}
\omega(a,x,y)=&\eta\frac{f(x,y)+((r+\delta)a-\tilde{c}^e)J_a(a,x,y)+(\rho-r)J(a,x,y)-\rho V(y)}{1-J_a(a,x,y)}\\
&-(1-\eta)\frac{u(\tilde{c}^e)+((r+\delta)a-\tilde{c}^e)W_a(a,x,y)-(\rho+\delta)U(a,x)}{W_a(a,x,y)}
\end{aligned}\tag{A1}$$

I.3 Additional Proofs

Proposition 6. $\lim_{\Delta\to 0}\tilde{c}^e_w(w,a,x,y;\Delta)=0$.

Proof. This is true because the optimal consumption policy is characterized by its first order condition

$$u'\left(\tilde{c}^e\right) = \frac{1}{1+\rho\Delta}\left\{\left(1-\Delta\sigma\right) W_a(a+\left(ra+w-\tilde{c}^e\right)\Delta, x, y) + \Delta\sigma U_a\left(a+\left(ra+w-\tilde{c}^e\right)\Delta, x\right)\right\}.$$

Notice that as $\Delta \to 0$, the limiting FOC becomes $\lim_{\Delta\to 0} u'\left(\tilde{c}^e\right) = W_a(a,x,y)$. Under mild technical conditions so that we can switch the order of limit and derivative,

$$\lim_{\Delta\to 0}\frac{\partial \tilde{c}^e}{\partial w}\left(w,a,x,y;\Delta\right) = \frac{\partial}{\partial w}\lim_{\Delta\to 0}\tilde{c}^e\left(w,a,x,y;\Delta\right) = \frac{\partial}{\partial w}u'^{(-1)}\left(W_a(a,x,y)\right) = 0.$$

□

I.4 Proof of Proposition 2

Proof. From the discussion before, we know that Nash bargaining implies the following relationship for the adjusted match surplus could be written as

$$\frac{W\left(a,x,y\right) - U\left(a,x\right)}{W_a\left(a,x,y\right)} = \eta\hat{S}\left(a,x,y\right).$$

Worker optimization gives rise to the first order condition $W_a\left(a,x,y\right) = \mathfrak{u}'\left(c^e\left(a,x,y\right)\right) > 0$. Therefore, whether a match is formed or not, i.e., whether $\hat{S}\left(a,x,y\right) > 0$ is equivalent to whether $W(a,x,y) - U\left(a,x\right) > 0$.

Consider a such that $W(a,x,y) - U\left(a,x\right) = 0$, i.e., a marginally acceptable match. Define $\Delta\left(a;x,y\right) \coloneqq W(a,x,y) - U\left(a,x\right)$. Differentiate both sides with respect to wealth a:

$$\Delta_a = W_a - U_a = \mathfrak{u}'\left(c^e\right) - \mathfrak{u}'\left(c^u\right),$$

where the arguments are suppressed for simplicity. It is obvious that for acceptable matches, $c^e > c^u$. Since the flow utility exhibits the usual concavity property $\mathfrak{u}'' < 0$, it must be that $\Delta_a = \mathfrak{u}'\left(c^e\right) - \mathfrak{u}'\left(c^u\right) < 0$.

Therefore, for any $a' > a$ we will have $\hat{S}\left(a',x,y\right) < 0$ and for any $a'' < a$ we will have $\hat{S}\left(a'',x,y\right) > 0$. □

I.5 Proof of Proposition 3

Proof. Consider $a > a'$. From Proposition 2 we know that $\Phi(a,x,y) \subset \Phi(a',x,y)$. Therefore the job finding rate of the worker of type x with wealth a is

$$\begin{aligned}\pi_{ue}(a,x) &= p(\theta)\int \frac{d_v(y)}{v}\Phi(a,x,y)\,\mathrm{d}y \\ &\leq p(\theta)\int \frac{d_v(y)}{v}\Phi(a',x,y)\,\mathrm{d}y \\ &= \pi_{ue}(a',x)\end{aligned}$$

□

I.6 Proof of Proposition 5

Proof. Total differentiating $W_a(a,x,y)$, we have

$$\mathrm{d}W_a(a,x,y) = W_{aa}(a,x,y)\,\mathrm{d}t$$

Apply the Envelope theorem to employed value $W(a,x,y)$ with respect to a,

$$\rho W_a(a,x,y) = \sigma\left[U_a(a,x) - W_a(a,x,y)\right] + \dot{a}W_{aa}(a,x,y) + \left[r + \omega_a(a,x,y)\right]W_a(a,x,y).$$

Note that $W_a(a,x,y) = u'(c^e(a,x,y))$ and $U_a(a,x) = u'(c^u(a,x))$ by FOCs

$$u''(c^e)\,\mathrm{d}c^e = (\rho - r - \omega_a)\,u'(c^e)\,\mathrm{d}t - \sigma\left[u'(c^u) - u'(c^e)\right]\mathrm{d}t$$

Rearrange

$$\underbrace{-\frac{u''(c^e)\,c^e}{u'(c^e)}}_{\text{relative risk aversion}} \cdot \underbrace{\frac{\mathrm{d}c^e/\,\mathrm{d}t}{c^e}}_{\text{consumption growth}} = r - \rho + \omega_a + \sigma\left[\frac{u'(c^u)}{u'(c^e)} - 1\right]$$

Similarly, total differentiating $U_a(a,x)$, we have

$$\mathrm{d}U_a(a,x) = U_{aa}(a,x)\left[ra + b - c^u\right]\mathrm{d}t$$

Apply the Envelope theorem to unemployed value $U(a,x)$ with respect to a

$$\rho U_a(a,x) = p(\theta)\int \frac{d_v(y)}{v}\left[W_a(a,x,y) - U_a(a,x)\right]^+ \mathrm{d}y + \dot{a}U_{aa}(a,x) + rU_a(a,x)$$

Plugging in FOCs

$$u''(c^u)\,\mathrm{d}c^u = (\rho - r)\,u'(c^u)\,\mathrm{d}t - p(\theta)\int_{B(a,x)} \frac{d_v(y)}{v}\left[u'(c^e) - u'(c^u)\right]\mathrm{d}y\,\mathrm{d}t$$

Rearrange

$$-\frac{u''(c^u)\,c}{u'(c^u)}\cdot\frac{\mathrm{d}c^u/\,\mathrm{d}t}{c} = r - \rho + p(\theta)\int_{B(a,x)} \frac{d_v(y)}{v}\left[\frac{u'(c^e)}{u'(c^u)} - 1\right]\mathrm{d}y$$

□

II Algorithmic Appendix

II.1 HJB Equations

Rewrite $W(a,x,y)$ as the employed value, and $U(a,x)$ as the unemployed value. The HJB equations are $(\rho+\delta)W(w,a,x,y) -> c_w$

$$\begin{aligned}
(\rho+\delta)W(a,x,y) &= \max_c u(c) + \sigma\left[U(a,x) - W(a,x,y)\right] + \tilde{\sigma}\left[U(a,x) - W(a,x,y)\right]^+ \\
&\quad + \left(ra + \omega(a,x,y) - c\right)W_a(a,x,y) \\
(\rho+\delta)U(a,x) &= \max_c u(c) + p(\theta)\sum_k \frac{d_v(k)}{v}\left[W(a,x,y_k) - U(a,x)\right]^+ \\
&\quad + \left(ra + b(x) - c\right)U_a(a,x)
\end{aligned}$$

with the first order conditions $u'(c) = W_a(a,x,y)$ and $u'(c) = U_a(a,x)$ respectively. The FD approximation to the HJB equations are

$$\begin{aligned}
(\rho+\delta)W(a_i,x_j,y_k) &= u(c_{i,j,k}) + \sigma\left[U(a_i,x_j) - W(a_i,x_j,y_k)\right] + \tilde{\sigma}\left[U(a_i,x_j) - W(a_i,x_j,y_k)\right]^+ \\
&\quad + \left(ra_i + \omega(a_i,x_j,y_k) - c_{i,j,k}\right)W_{a,}(a_i,x_j,y_k) \qquad \text{(A2)} \\
(\rho+\delta)U(a_i,x_j) &= u(c_{i,j}) + p(\theta)\sum_k \frac{d_v(k)}{v}\left[W(a_i,x_k,y_k) - U(a_i,x_j)\right]^+ \\
&\quad + \left(ra_i + b(x_j) - c\right)U_a(a_i,x_k) \qquad \text{(A3)}
\end{aligned}$$

Similarly, write producing firms' HJB equations as

$$\begin{aligned} rJ(a_i, x_j, y_k) = f(x_i, y_k) - \omega(a_i, x_j, y_k) + (\sigma + \delta)\left[V(y_k) - J(a_i, x_j, y_k)\right] \\ + \tilde{\sigma}\left[V(y_k) - J(a_i, x_j, y_k)\right]^+ + \left(ra_i + \omega\left(a_i, x_j, y_k\right) - c\right) J_a(a_i, x_j, y_k) \end{aligned} \tag{A4}$$

and vacant firms' HJB equations as

$$rV(y_k) = q(\theta) \sum_i \sum_j \frac{d_u(a_i, x_j)}{u}\left[J(a_i, x_j, y_k) - V(y_k)\right]^+ \tag{A5}$$

II.2 Upwind Scheme

To compute the HJB equations, we need to approximate the derivatives of value functions numerically. Here we follow Achdou et al. (2022) and use the upwind scheme. The idea is to basically use the forward difference approximation whenever savings policy is positive, and backward difference whenever savings is negative.

Define the forward difference and backward difference as

$$W_{a,F}\left(a_i, x_j, y_k\right) = \frac{W\left(a_{i+1}, x_j, y_k\right) - W\left(a_i, x_j, y_k\right)}{\Delta_a}$$

$$W_{a,B}\left(a_i, x_j, y_k\right) = \frac{W\left(a_i, x_j, y_k\right) - W\left(a_{i-1}, x_j, y_k\right)}{\Delta_a}$$

$$\bar{W}_a\left(a_i, x_j, y_k\right) = u'\left(ra_i + w\left(a_i, x_j, y_k\right)\right)$$

We use the "upwind scheme". From the first order condition we can get $c = \left(u'\right)^{-1} W_a\left(a, x, y\right)$. Define

$$s^W_{i,j,k,F} = ra_i + w\left(a_i, x_j, y_k\right) - \left(u'\right)^{-1}\left(W_{a,F}\left(a_i, x_j, y_k\right)\right)$$

$$s^W_{i,j,k,B} = ra_i + w\left(a_i, x_j, y_k\right) - \left(u'\right)^{-1}\left(W_{a,B}\left(a_i, x_j, y_k\right)\right)$$

and approximate the derivative as follows

$$\begin{aligned} W_a\left(a_i, x_j, y_k\right) = W_{a,B}\left(a_i, x_j, y_k\right)\mathbf{1}_{\left\{s^W_{i,j,k,B}<0\right\}} + W_{a,F}\left(a_i, x_j, y_k\right)\mathbf{1}_{\left\{s^W_{i,j,k,F}>0\right\}} \\ + \bar{W}_a\left(a_i, x_j, y_k\right)\mathbf{1}_{\left\{s^W_{i,j,k,F}<0<s^W_{i,j,k,B}\right\}}. \end{aligned} \tag{A6}$$

The derivative of firms' HJB J_a is approximated in the same way.

Since W is concave in a, we have $s^W_{i,j,k,F} < s^W_{i,j,k,B}$, then at some point i we have $s^W_{i,j,k,F} < 0 < s^W_{i,j,k,B}$, in which case we set savings to 0. Plugging the expression (A6) into the discretized

HJB equation (A2), then the HJB equation can be written as

$$
\begin{aligned}
(\rho+\delta) W_i^{jk} = u\left(c_i^{jk}\right) + \sigma\left[U_i^j - W_i^{jk}\right] + \tilde{\sigma}\left[U_i^j - W_i^{jk}\right]^+ \\
+ \underbrace{\frac{W_{i+1}^{jk} - W_i^{jk}}{\Delta_a}}_{W_{a,F}} s_{i,F}^{jk,W+} + \underbrace{\frac{W_i^{jk} - W_{i-1}^{jk}}{\Delta_a}}_{W_{a,B}} s_{i,B}^{jk,W-}
\end{aligned}
$$

Let $\sigma_i^{jk} = \sigma$ if no endogenous separation, and $\sigma_i^{jk} = \sigma + \tilde{\sigma}$ otherwise. In matrix notation

$$
\begin{aligned}
(\rho+\delta) W_i^{jk} &= u\left(c_i^{jk}\right) + \sigma_i^{jk}\left[U_i^j - W_i^{jk}\right] \\
&+ \frac{1}{\Delta_a}\left[\begin{array}{ccc} -s_{i,B}^{jk,W-}, & s_{i,B}^{jk,W-} - s_{i,F}^{jk,W+}, & s_{i,F}^{jk,W+} \end{array}\right]\left[\begin{array}{c} W_{i-1}^{jk} \\ W_i^{jk} \\ W_{i+1}^{jk} \end{array}\right]
\end{aligned} \tag{A7}
$$

Similarly define

$$
\begin{aligned}
(\rho+\delta) U_i^j &= u\left(c_i^j\right) + p(\theta) \sum_k \frac{d_v(k)}{v}\left[W_i^{jk} - U_i^j\right]^+ \\
&+ \frac{1}{\Delta_a}\left[\begin{array}{ccc} -s_{i,B}^{j,U-}, & s_{i,B}^{j,U-} - s_{i,F}^{j,U+}, & s_{i,F}^{j,U+} \end{array}\right]\left[\begin{array}{c} U_{i-1}^j \\ U_i^j \\ U_{i+1}^j \end{array}\right]
\end{aligned} \tag{A8}
$$

and

$$
\begin{aligned}
r J_i^{jk} &= f^{jk} - \omega_i^{jk} + \left(\sigma_i^{jk} + \delta\right)\left[V^k - J_i^{jk}\right] \\
&+ \frac{1}{\Delta_a}\left[\begin{array}{ccc} -s_{i,B}^{jk,W-}, & s_{i,B}^{jk,W-} - s_{i,F}^{jk,W+}, & s_{i,F}^{jk,W+} \end{array}\right]\left[\begin{array}{c} J_{i-1}^{jk} \\ J_i^{jk} \\ J_{i+1}^{jk} \end{array}\right]
\end{aligned} \tag{A9}
$$

In some cases it would be more efficient to use non-uniform grids, especially for wealth, as the distribution tends to be right-skewed, with the mode being at a low wealth level. There are many ways to implement non-uniform grids. Here we show one example. Let a_{min} and a_{max} denote the minimum and maximum wealth levels in the grid respectively. The grid points are $a_i = a_{min} + \frac{q^{x_i}-1}{q-1}$ where $q > 1$ and x_i, $i = 1, ..., N_a$ is a uniform grid on $[0, 1]$. The intervals between grid points can then be defined as $\Delta_{a,i}^F \equiv a_{i+1} - a_i$ and $\Delta_{a,i}^B \equiv a_i - a_{i-1}$. We can then use $\Delta_{a,i}^F$ in place of Δ_a whenever forward difference is applied and $\Delta_{a,i}^B$ in place of Δ_a whenever backward difference is applied.

II.3 Implicit method

Let $\mathbf{W}$ denote the vector that stacks all value functions together. The implicit method updates the value functions in the following way:

$$\frac{1}{\Delta}\left(\mathbf{W}^{n+1}-\mathbf{W}^{n}\right)+(\rho+\delta)\mathbf{W}^{n+1}=\tilde{\mathbf{u}}\left(\mathbf{W}^{n}\right)+\mathbf{A}\left(\mathbf{W}^{n}\right)\mathbf{W}^{n+1}$$

which gives

$$\left(\left(\rho+\delta+\frac{1}{\Delta}\right)\mathbf{I}-\mathbf{A}\left(\mathbf{W}^{n}\right)\right)\mathbf{W}^{n+1}=\tilde{\mathbf{u}}\left(\mathbf{W}^{n}\right)+\frac{1}{\Delta}\mathbf{W}^{n}$$

$$\Rightarrow \mathbf{W}^{n+1}=\left(\left(\rho+\delta+\frac{1}{\Delta}\right)\mathbf{I}-\mathbf{A}\left(\mathbf{W}^{n}\right)\right)^{-1}\left(\tilde{\mathbf{u}}\left(\mathbf{W}^{n}\right)+\frac{1}{\Delta}\mathbf{W}^{n}\right)$$

Stack the value $\mathbf{W}$ where we first loop over assets $a_1,...,a_{Na}$, then over worker skills $x_1,..,x_{N_x}$, and then finally over firm type $y_1,...,y_{N_y}$ in the outer loop. In particular, let

$$\mathbf{W}=\begin{pmatrix}\mathbf{W}_1^{11}\\ \mathbf{W}_2^{11}\\ \vdots\\ \mathbf{W}_{N_a}^{11}\\ \mathbf{W}_1^{21}\\ \vdots\\ \mathbf{W}_{N_a}^{N_xN_y}\\ \mathbf{U}_1^{1}\\ \mathbf{U}_2^{1}\\ \vdots\\ \mathbf{U}_{N_a}^{1}\\ \mathbf{U}_1^{2}\\ \vdots\\ \mathbf{U}_{N_a}^{N_x}\end{pmatrix}$$

The matrix $\mathbf{A}\left(\mathbf{W}^{n}\right)$ has three components: one with respect to asset accumulation (the last terms of equations (A7) and (A8)), another with respect to job separation $\sigma_i^{jk}\left[U_i^j-W_i^{jk}\right]$, and the last one with respect to job matching $p\left(\theta\right)\sum_k\frac{d_v(k)}{v}\left[W_i^{jk}-U_i^j\right]^+$, which we denote as $\mathbf{A}_1$, $\mathbf{A}_2$ and $\mathbf{A}_3$ respectively, then $\mathbf{A}\left(\mathbf{W}^{n}\right)=\mathbf{A}_1+\mathbf{A}_2+\mathbf{A}_3$ such that

$$\mathbf{A}_1=\begin{bmatrix}\mathbf{A}_{1e} & 0\\ 0 & \mathbf{A}_{1u}\end{bmatrix}$$

$$
\mathbf{A}_{1e}=\begin{bmatrix}
\beta_1^{11,W} & \gamma_1^{11,W} & 0 & \cdots & 0 \\
\alpha_2^{11,W} & \beta_2^{11,W} & \gamma_2^{11,W} & 0 & \cdots \\
0 & \alpha_3^{11,W} & \beta_3^{11,W} & \gamma_3^{11,W} & 0 \\
\vdots & \ddots & \ddots & \ddots & \ddots \\
0 & \ddots & \ddots & \alpha_{Na}^{NxNy,W} & \beta_{Na}^{N_xN_y,W}
\end{bmatrix},\mathbf{A}_{1u}=\begin{bmatrix}
\beta_1^{1,U} & \gamma_1^{1,U} & 0 & \cdots & 0 \\
\alpha_2^{1,U} & \beta_2^{1,U} & \gamma_2^{1,U} & 0 & 0 \\
0 & \alpha_3^{1,U} & \beta_3^{1,U} & \gamma_3^{1,U} & 0 \\
\ddots & \ddots & \ddots & \ddots & \ddots \\
0 & \cdots & 0 & \beta_{Na}^{N_x,U} & \gamma_{N_a}^{N_x,U}
\end{bmatrix}
$$

where

$$
\alpha_i^{jk,W}=\frac{-s_{i,B}^{jk,W-}}{\Delta_a},\quad \beta_i^{jk,W}=\frac{s_{i,B}^{jk,W-}-s_{i,F}^{jk,W+}}{\Delta_a},\quad \gamma_i^{jk,W}=\frac{s_{i,F}^{jk,W+}}{\Delta_a},
$$

and analogously for the unemployed coefficients.

$$
\mathbf{A}_2=\begin{bmatrix}
-\sigma_1^{11} & 0 & \cdots & 0 & & & & & & & & & 0 & \sigma_1^{11} & 0 & \cdots & 0 \\
0 & -\sigma_2^{11} & 0 & 0 & & & & & & & & & \vdots & 0 & \sigma_2^{11} & 0 & 0 \\
\vdots & \ddots & \ddots & \vdots & & & & & & & & & \vdots & \vdots & \ddots & \ddots & \vdots \\
0 & 0 & \cdots & -\sigma_{N_a}^{11} & & & & & & & & & \vdots & 0 & \cdots & 0 & \sigma_{N_a}^{11} \\
\vdots & \ddots & \ddots & \ddots & \ddots & & & & & & & & \vdots & \sigma_1^{21} & 0 & \cdots & 0 \\
\vdots & & & & & \ddots & & & & & & & \vdots & 0 & \sigma_2^{21} & 0 & 0 \\
\vdots & & & & & & \ddots & & & & & & \vdots & \vdots & \ddots & \ddots & \vdots \\
\vdots & & & & & & & \ddots & & & & & \vdots & 0 & \cdots & 0 & \sigma_{N_a}^{21} \\
\vdots & & & & & & & & & \ddots & & & \vdots & \sigma_1^{N_xN_y} & 0 & \cdots & 0 \\
\vdots & & & & & & & & & & \ddots & & \vdots & 0 & \sigma_2^{N_xN_y} & 0 & 0 \\
\vdots & & & & & & & & & & & \ddots & 0 & \vdots & \ddots & \ddots & \vdots \\
0 & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & 0 & -\sigma_{N_a}^{N_xN_y} & 0 & \cdots & 0 & \sigma_{N_a}^{N_xN_y} \\
0 & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & 0 \\
\vdots & \ddots & \ddots & \ddots & \ddots & \ddots & \ddots & \ddots & \ddots & \ddots & \ddots & \ddots & \ddots & \ddots & \ddots & \ddots & \vdots \\
0 & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & 0
\end{bmatrix}
$$

where each diagonal submatrix corresponds to a loop over asset states $a_1, ..., a_{N_a}$ and worker skills $x_1, ..., x_{N_x}$. The bottom part is a matrix of $N_1 \times N_2$ zeros where $N_1 = N_a \times N_x$ and $N_2 = N_a \times N_x \times (N_y + 1)$.

$$
\mathbf{A}_3=p\left(\theta\right)\times\begin{bmatrix}
0 & \cdots & \cdots & \cdots & 0 \\
\vdots & \ddots & \ddots & \ddots & \vdots \\
0 & \cdots & \cdots & \cdots & 0 \\
\mathbf{A}_{31} & \mathbf{A}_{32} & \cdots & \mathbf{A}_{3N_y} & \mathbf{A}_{3N_y+1}
\end{bmatrix}
$$

where

$$\mathbf{A}_{3k}=\begin{bmatrix} d_v^k\mathbb{1}_1^{1k} & 0 & \cdots & \cdots & \cdots & 0 \\ 0 & d_v^k\mathbb{1}_2^{1k} & \ddots & \ddots & \ddots & \vdots \\ \vdots & \ddots & \ddots & \ddots & \ddots & \vdots \\ \vdots & \ddots & \ddots & d_v^k\mathbb{1}_{N_a}^{1k} & \ddots & \vdots \\ \vdots & \ddots & \ddots & \ddots & \ddots & \vdots \\ 0 & \cdots & \cdots & \cdots & 0 & d_v^k\mathbb{1}_{N_a}^{N_xk} \end{bmatrix}$$

$$\mathbb{1}_i^{jk}=\begin{cases} 1 & \text{if } W\left(a_i,x_j,y_k\right)>U\left(a_i,x_j\right) \\ 0 & \text{otherwise} \end{cases}$$

and

$$\mathbf{A}_{3N_y+1}=\begin{bmatrix} -\sum_k d_v^k\mathbb{1}_1^{1k} & 0 & \cdots & \cdots & \cdots & 0 \\ 0 & -\sum_k d_v^k\mathbb{1}_2^{1k} & \ddots & \ddots & \ddots & \vdots \\ \vdots & \ddots & \ddots & \ddots & \ddots & \vdots \\ \vdots & \ddots & \ddots & -\sum_k d_v^k\mathbb{1}_{N_a}^{1k} & \ddots & \vdots \\ \vdots & \ddots & \ddots & \ddots & \ddots & \vdots \\ 0 & \cdots\cdots & \cdots\cdots & \cdots\cdots & 0 & -\sum_k d_v^k\mathbb{1}_{N_a}^{N_xk} \end{bmatrix}$$

and the top part is a matrix of $N_1 \times N_2$ zeros where $N_1 = N_a \times N_x \times N_y$ and $N_2 = N_a \times N_x \times (N_y + 1)$.

Similarly, let $\mathbf{J}$ denote the vector that stacks all firm value functions together, and $\mathbf{\Pi}$ denote the vector of firm flow profits. The implicit method updates firms' value functions in the following way

$$\frac{1}{\Delta}\left(\mathbf{J}^{n+1}-\mathbf{J}^{n}\right)+r\mathbf{J}^{n+1}=\mathbf{\Pi}+\mathbf{B}\left(\mathbf{W}\right)\mathbf{J}^{n+1}$$

The matrix $\mathbf{B}\left(\mathbf{W}\right)$ also has three components: one with respect to workers' asset accumulation (hence the dependence on $\mathbf{W}$), another with respect to separation $\left(\sigma_i^{jk}+\delta\right)\left[V^j-J_i^{jk}\right]$, and the last one with respect to worker matching $q(\theta)\sum_i\sum_j\frac{d_u(i,j)}{u}\left[J_i^{jk}-V^j\right]^+$. Note that the first component is exactly the same as $\mathbf{A}_1$, so we get it for free once we have finished workers' system of PDEs.

The next two components can be constructed in an analogous way to $\mathbf{A}_2$ and $\mathbf{A}_3$ and we therefore omit the details.

II.4 Stationary Density

Recall the Kolmogorov Forward (KF) equations for density:

$$
\begin{aligned}
0 =& -\frac{\partial}{\partial a}\left[s^{W}(a,x,y)\,d_m(a,x,y)\right] - \{\sigma+\delta+\tilde{\sigma}[1-\Phi(a,x,y)]\}d_m(a,x,y) \\
&+ p(\theta)\frac{d_v(y)}{v}\Phi(a,x,y)\,d_u(a,x) \\
0 =& -\frac{\partial}{\partial a}\left[s^{U}(a,x)\,d_u(a,x)\right] - \int p(\theta)\frac{d_v(y)}{v}\Phi(a,x,y)\,d_u(a,x)\,\mathrm{d}y - \delta d_u(a,x) \\
&+ \sigma\int d_m(a,x,y)\,\mathrm{d}y + \tilde{\sigma}\int[1-\Phi(a,x,y)]\,d_m(a,x,y)\,\mathrm{d}y + \delta d_x\cdot\mathbb{1}\{a=0\}
\end{aligned}
$$

together with the condition that density integrates to 1:

$$
1 = \int_{\underline{a}}^{\infty} d_m(a,x,y)\,\mathrm{d}a\,\mathrm{d}x\,\mathrm{d}y + \int_{\underline{a}}^{\infty} d_u(a,x)\,\mathrm{d}a\,\mathrm{d}x
$$

as well as

$$
\begin{aligned}
d_x &= \int_{\underline{a}}^{\infty} d_m(a,x,y)\,\mathrm{d}a\,\mathrm{d}y + \int_{\underline{a}}^{\infty} d_u(a,x)\,\mathrm{d}a \\
d_y &= \int_{\underline{a}}^{\infty} d_m(a,x,y)\,\mathrm{d}a\,\mathrm{d}x + d_v(y)
\end{aligned}
$$

which can be discretized as

$$
\begin{aligned}
0 &= -\frac{\partial}{\partial a}\left[s_i^{jk,W}d_i^{jk,W}\right] - \left(\sigma_i^{jk}+\delta\right)d_i^{jk,W} + p(\theta)\frac{d_v^k}{v}\mathbb{1}_i^{jk}d_i^{j,U} \\
0 &= -\frac{\partial}{\partial a}\left[s_i^{j,U}d_i^{j,U}\right] - p(\theta)\sum_{k=1}^{N_y}\frac{d_v^k}{v}\mathbb{1}_i^{jk}d_i^{j,U} + \sigma_i^{jk}\sum_{k=1}^{N_y}d_i^{jk,W} + \delta\mathbb{1}\{i=1\}\sum_{i'}\left(d_{i'}^{j,U}+\sum_k d_{i'}^{jk,W}\right)
\end{aligned}
$$

and

$$
\begin{aligned}
1 &= \sum_{i=1}^{N_a}\sum_{j=1}^{N_x}\sum_{k=1}^{N_y} d_i^{jk,W}\Delta_a\Delta_x\Delta_y + \sum_{i=1}^{N_a}\sum_{j=1}^{N_x} d_i^{j,U}\Delta_a\Delta_x \qquad \text{(A10)}\\
d_x^j &= \sum_{i=1}^{N_a}\sum_{k=1}^{N_y} d_i^{jk,W}\Delta_a\Delta_y + \sum_{i=1}^{N_a} d_i^{j,U}\Delta_a \\
d_y^k &= \sum_{i=1}^{N_a}\sum_{j=1}^{N_x} d_i^{jk,W}\Delta_a\Delta_x + d_v^k
\end{aligned}
$$

II.5 Upwind Scheme

For the derivatives, we again use the forward scheme

$$
\begin{aligned}
0 = & -\frac{d_i^{jk,W} s_{i,F}^{jk,W+} - d_{i-1}^{jk,W} s_{i-1,F}^{jk,W+}}{\Delta_a} - \frac{d_{i+1}^{jk,W} s_{i+1,B}^{jk,W-} - d_i^{jk,W} s_{i,B}^{jk,W-}}{\Delta_a} \\
& - \left(\sigma_i^{jk} + \delta\right) d_i^{jk,W} + p\left(\theta\right) \frac{d_v^k}{v} \mathbb{1}_i^{jk} d_i^{j,U} \\
0 = & -\frac{d_i^{j,U} s_{i,F}^{j,U+} - d_{i-1}^{j,U} s_{i-1,F}^{j,U+}}{\Delta_a} - \frac{d_{i+1}^{j,U} s_{i+1,B}^{j,U-} - d_i^{j,U} s_{i,B}^{j,U-}}{\Delta_a} - p\left(\theta\right) \sum_{k=1}^{N_y} \frac{d_v^k}{v} \mathbb{1}_i^{jk} d_i^{j,U} + \sigma_i^{jk} \sum_{k=1}^{N_y} d_i^{jk,W} \\
& + \delta \mathbb{1}\{i=1\} \sum_{i'} \left(d_{i'}^{j,U} + \sum_k d_{i'}^{jk,W} \right)
\end{aligned}
$$

Collecting terms, we have

$$
\begin{aligned}
0 &= d_{i-1}^{jk,W} \alpha_{i-1}^{jk,W} + d_i^{jk,W} \beta_i^{jk,W} + d_{i+1}^{jk,W} \gamma_{i+1}^{jk,W} + p\left(\theta\right) \frac{d_v^k}{v} \mathbb{1}_i^{jk} d_i^{j,U} \\
0 &= d_{i-1}^{j,U} \alpha_{i-1}^{j,U} + d_i^{j,U} \beta_i^{j,U} + d_{i+1}^{j,U} \gamma_{i+1}^{j,U} + \sigma_i^{jk} \sum_{k=1}^{N_y} d_i^{jk,W} + \delta \mathbb{1}\{i=1\} \sum_{i'} \left(d_{i'}^{j,U} + \sum_k d_{i'}^{jk,W} \right)
\end{aligned}
$$

where

$$
\begin{cases}
\alpha_{i-1}^{jk,W} & = \frac{s_{i-1,F}^{jk,W+}}{\Delta_a} \\
\beta_i^{jk,W} & = \frac{s_{i,B}^{jk,W-} - s_{i,F}^{jk,W+}}{\Delta_a} - \delta \\
\gamma_i^{jk,W} & = -\frac{s_{i+1}^{jk,W-}}{\Delta_a}
\end{cases}
\qquad
\begin{cases}
\alpha_{i-1}^{j,U} & = \frac{s_{i-1,F}^{j,U+}}{\Delta_a} \\
\beta_i^{j,U} & = \frac{s_{i,B}^{j,U-} - s_{i,F}^{j,U+}}{\Delta_a} - p\left(\theta\right) \sum_{k=1}^{N_y} \frac{d_v^k}{v} \mathbb{1}_i^{jk} \\
\gamma_i^{j,U} & = -\frac{s_{i+1}^{j,U-}}{\Delta_a}
\end{cases}
$$

Let $\mathbf{d}$ be the stacked vector of densities (arranged in the same order as $\mathbf{W}$), then the KF equations expressed using the upwind scheme can be written as

$$
\tilde{\mathbf{A}} \mathbf{d} = 0 \tag{A11}
$$

where $\tilde{\mathbf{A}}$ is related to the transpose of $\mathbf{A}$, the matrix that was defined in the implicit method in Section II.3. Following the construction of $\mathbf{A}$, we can also write $\tilde{\mathbf{A}}$ as the sum of a component due to asset accumulation, a component due to job separation (and exiting the economy), and a component due to job finding. Note that unlike the setup in Achdou et al. (2022) where the KF equations are directly the transpose of the HJB equations, here we need to rewrite the job separation and exiting component of the matrix as the exit shocks (which happens at Poisson rate δ) leave workers with 0 continuation value so is in essence equivalent to increasing workers' discount rate ρ by δ, while for the worker flow equations, the exit shocks reallocate workers to the unemployed state with 0 assets. To make the adjustment, simply subtract the diagonal elements of $\mathbf{A}_2$ by δ, increase the elements corresponding to the transition from employed state (i,j,k) and unemployed state (i,j) to $(1,j)$ by δ, and transpose the matrix.

To solve the problem of equation (A11) subject to the constraints (A10), we can do the following. Fix (1) either $d_i^{jk,W}$ or $d_i^{j,U}$ to be 0.1 (or any other non-zero number) for arbitrary (i,j,k); (2) then solve the system for some $\tilde{d}$ and then to renormalize

$$d_i^{jk,W} = \tilde{d}_i^{jk,W} / \left(\sum_{i=1}^{N_a} \sum_{j=1}^{N_x} \sum_{k=1}^{N_y} \tilde{d}_i^{jk,W} \Delta_a \Delta_x \Delta_y + \sum_{i=1}^{N_a} \sum_{j=1}^{N_x} \tilde{d}_i^{j,U} \Delta_a \Delta_x \right)$$

and

$$d_i^{j,U} = \tilde{d}_i^{j,U} / \left(\sum_{i=1}^{N_a} \sum_{j=1}^{N_x} \sum_{k=1}^{N_y} \tilde{d}_i^{jk,W} \Delta_a \Delta_x \Delta_y + \sum_{i=1}^{N_a} \sum_{j=1}^{N_x} \tilde{d}_i^{j,U} \Delta_a \Delta_x \right).$$

III Data and Empirics Appendix

III.1 Construction of Worker and Firm Types

This section describes the method to construct multi-dimensional worker skills and job skill requirements, used by Lise and Postel-Vinay (2020). We create 2-dimensional worker skill bundles and job skill requirement bundles using a data set combining NLSY79 job history and O*NET, following Lise and Postel-Vinay (2020). For jobs, we

- match weekly NLSY79 job record to O*NET data which contains measures of a variety of job skill descriptors
- take the first 2 principal components of these measures in the panel
- recombine the 2 principal components so that they satisfy the following exclusion restrictions: (1) the *mathematics* measure only reflects cognitive skill requirements; (2) the *mechanical knowledge* scores only reflects manual skill requirements
- normalize the skill requirements so that each component lies in $[0, 1]$

For workers, we

- use all 10 components of individual ASVAB test scores and a measure of health (BMI)
- take the first 2 principal components of these measures
- recombine them so that (1) the ASVAB *mathematics knowledge* score only reflects cognitive skills; (2) the ASVAB *automotive and shop information* score only reflects manual skills
- normalize the skill measures so that each component lies in $[0, 1]$

In the analysis above, we only use the first component, i.e. cognitive skill/skill requirement.

III.2 Descriptive Statistics

III.2.1 Skill Measures and Sorting

Our selected sample comprises 3,303 individuals with a broad range of educational backgrounds, spanning from no degree to a Ph.D. It is expected that our measures of worker skills and job skill requirements will reflect their relative rankings and productivity. To test this prediction, we look at how these two measures vary with levels of education. Table A-1 displays average worker cognitive skills and job cognitive skill requirements by the highest degree at the time of initial labor market entry. Both measures are normalized to a unit-length range $[0, 1]$, where higher numbers signify higher cognitive skills or skill requirements.

Table A-1: Average Worker Cognitive Skills and Job Skill Requirements, by Level of Education

	High School	Some College	2-yr College	4-yr College	Masters	PhD
Worker Skill (x)	0.388	0.456	0.543	0.684	0.714	0.770
Job Skill Req (y)	0.272	0.304	0.336	0.425	0.474	0.504
Observations	1053	233	402	500	323	99

Notes: Both x and y are normalized to $[0, 1]$.

Comparison of the skill measures at the lowest and highest education levels (No Degree and Ph.D.) reveals that education appears to account for a considerable portion of worker skill heterogeneity, and a moderate portion of job skill heterogeneity. It is perhaps not surprising that both worker skills (first row) and job skill requirements (second row) increase monotonically with education. Thus, at least in terms of skill differences across education groups, our skill measures effectively capture the relative ranking of workers and jobs, as well as positive assortative matching. However, outstanding questions include whether we can identify sorting beyond education using the cognitive skill measures, and whether positive assortative matching remains after controlling for education. To answer these questions, we present the correlation between job skill requirements and worker skills in Table A-2, both with and without controlling for worker education.

Table A-2: Skill Sorting Over Occupations

	Job Skill Req (y)	Job Skill Req (y)
Worker Skill (x)	0.691***	0.513***
	(0.020)	(0.027)
Education Level	No	Yes
Obs	35616	35616

Standard errors in parentheses, * p<0.10, ** p<0.05, *** p<0.01

Notes: Standard errors are clustered on occupation level. The sample is taken from the first observations of each worker-firm match.

We regress job cognitive skill requirements on worker cognitive skills. The first column of

Table A-2 shows that the correlation between worker skills and job skill requirements is 0.69, both statistically and economically significant. To isolate sorting on skills from sorting on education, we perform an additional regression controlling for dummies for years of education. After controlling for education, the remaining correlation is still large and significant at 0.51, suggesting a substantial amount of sorting on individual skills beyond sorting on education.

III.2.2 Initial Liquid Wealth and Worker Characteristics

There are 1,163 individuals with valid information about liquid wealth at the time when they entered the labor market. We define net liquid wealth as the value of financial assets such as cash, deposits, mutual funds, and money market accounts net of debts that are not asset-backed. This measure reflects assets that workers can access within a relatively short period. Table A-3 shows the characteristics of workers upon labor market entry, where the workers are divided into quintiles based on their liquid wealth during the first month of work.

Table A-3: Worker Characteristics by Initial Wealth Quintile

	Quintile 1	Quintile 2	Quintile 3	Quintile 4	Quintile 5
Net Financial Assets (1000s)	-7.971	0.346	1.764	5.356	31.83
Weekly Income	233.8	195.2	193.3	255.8	301.4
Years of Educ	15.91	14.54	15.38	15.88	16.25
Age	27.48	27.09	26.98	27.68	29.13
Male	0.416	0.405	0.368	0.446	0.350
PRTs Annual Income	19874.5	18343.5	23147.3	25479.3	25623.4
Observations	202	200	204	202	203

Notes: Liquid assets, weekly income and parents' annual income are in 1982 dollars.

The level of liquid wealth upon labor market entry exhibits substantial heterogeneity, ranging from $-7,971 in the lowest quintile to $31,830 in the highest quintile (in 1982 dollars), a difference of almost $40,000. Workers entering the labor market with higher liquid wealth tend to have higher income, more education, a higher age, and higher parental income. The only exception is the lowest quintile, where weekly income, years of education, age, and parental income are all higher than those in the quintile above. This counterintuitive result could be explained by the fact that the lowest liquid wealth quintile may include individuals who borrow substantial amount of debt for their higher education, thereby lowering their initial wealth. Note that for this result, the age of labor market entry is upward biased (most workers enter labor market in early 20s) because NLSY79 did not start collecting wealth information until 1985, when half of the sample were above 25.

III.3 Estimates of the Effect of Wealth on Labor Market Outcomes

Table A-4: Effect of Wealth on Mismatch

	Under-match			Over-match			Mismatch magnitude		
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
Liquid wealth ($1,000s)	-0.826***	-0.803***	-0.776***	0.249	0.280	0.331	-0.857***	-0.826***	-0.800***
	(0.133)	(0.133)	(0.133)	(0.471)	(0.473)	(0.474)	(0.133)	(0.133)	(0.133)
Demographic control	✓	✓	✓	✓	✓	✓	✓	✓	✓
Experience control		✓	✓		✓	✓		✓	✓
Economics control			✓			✓			✓
Observations	4706	4706	4706	1451	1451	1451	6157	6157	6157
R^2	0.256	0.260	0.265	0.0249	0.0303	0.0318	0.204	0.206	0.209

Standard errors in parentheses, * p<0.10, ** p<0.05, *** p<0.01

Notes: All coefficients and standard errors are multiplied by 100. Under-match means mismatch conditional on $y < x$, i.e. the worker is relatively more productive than the matched firm, while over-match means the opposite. Mismatch magnitude means the absolute value of mismatch $|y - x|$. Demographic controls include race, gender, education, ASVAB score, worker productivity x and a quadratic function of age. Experience controls include quadratic functions of work and occupation tenure. Economic controls include region fixed effect and national unemployment rate. Standard errors of coefficients are shown in parentheses.

Table A-5: Effect of Wealth on Labor Market Outcomes

	Job tenure (months)			Logged wages			Months unemployed		
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
Liquid wealth (\$1,000s)	0.391***	0.384***	0.341***	0.011***	0.010***	0.010***	0.330***	0.384***	0.364***
	(0.067)	(0.067)	(0.067)	(0.001)	(0.001)	(0.001)	(0.081)	(0.081)	(0.081)
Demographic control	✓	✓	✓	✓	✓	✓	✓	✓	✓
Experience control		✓	✓		✓	✓		✓	✓
Economics control			✓			✓			✓
Observations	7068	7068	4706	6791	6791	6791	8202	8202	8202
R^2	0.034	0.041	0.060	0.178	0.198	0.202	0.034	0.039	0.045

Standard errors in parentheses, * p<0.10, ** p<0.05, *** p<0.01

Notes: Demographic controls include race, gender, education, ASVAB score, worker productivity x and a quadratic function of age. Experience controls include quadratic functions of work and occupation tenure. Economic controls include region fixed effect and national unemployment rate. Standard errors of coefficients are shown in parentheses.

IV Quantitative Results Appendix

IV.1 Production Functions

Production Function with Vertical Heterogeneity

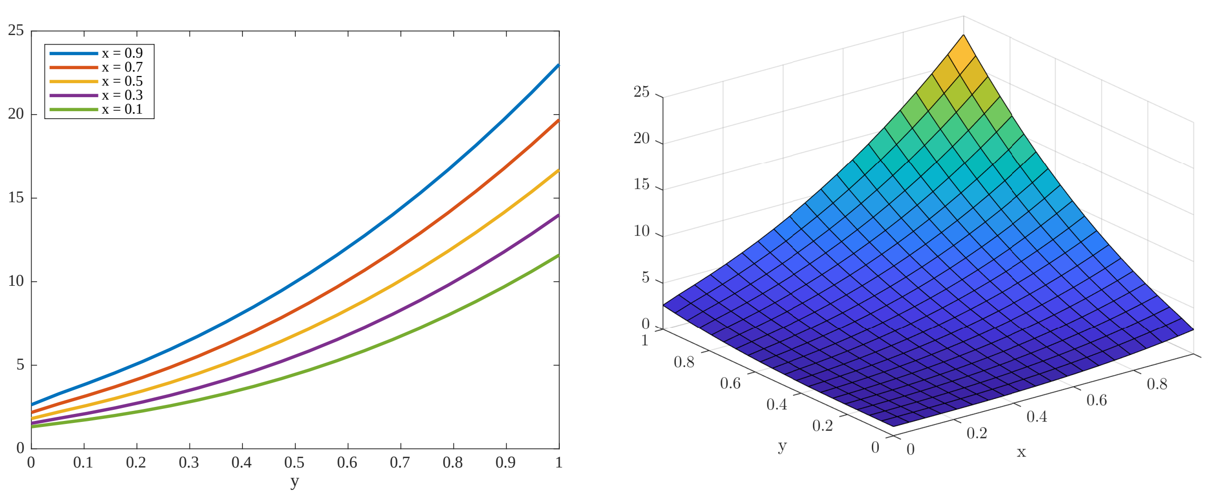
The production is specified by a CES function:

$$f(x, y) = f_0 + f_1 \left(x^\xi + y^\xi\right)^{\nu/\xi}, \quad x, y \in [0, 1]$$

where f_0 controls the level of minimum production, f_1 controls the scale, ξ controls the elasticity of substitution between worker productivity x and firm productivity y, and ν controls the curvature of the production function.

The production function is illustrated in Figure A-1.

Figure A-1: Production Function $f(x, y) = f_0 + f_1 \left(x^\xi + y^\xi\right)^{\nu/\xi}$



Notes: This graph illustrates the production function with vertical heterogeneity, specified by the CES form. The left panel shows the function with respect to y at different levels of x. The right panel shows the function in 3-D format.

Shimer and Smith (2000) shows that the search equilibrium features positive assortative matching if the production function is log-supermodular. This condition is satisfied if we assume $\xi \in (0, 1)$.

Production Function with Horizontal Heterogeneity

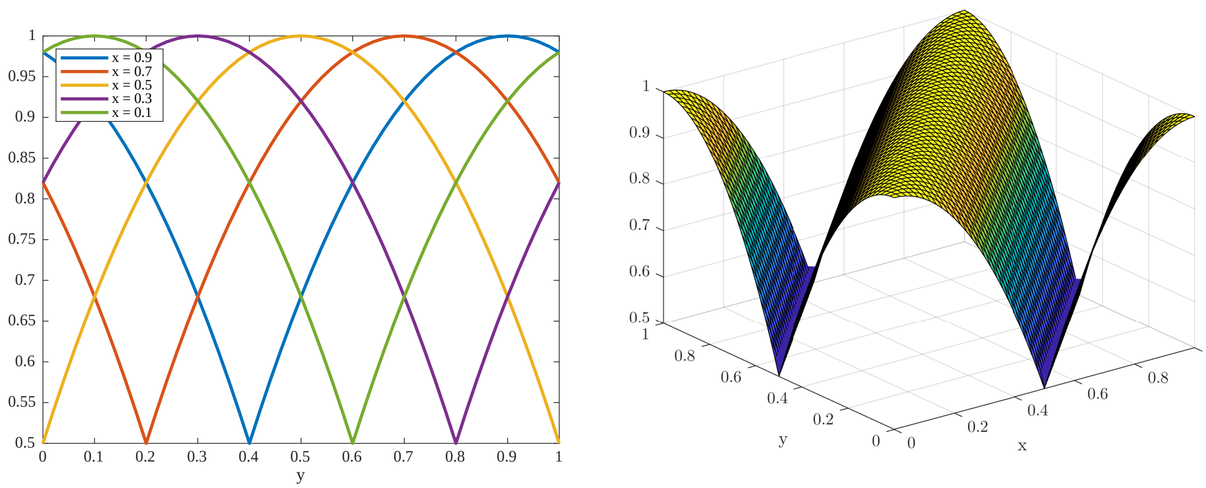
The production function is specified by the following form:

$$f(x,y) = a - b\min\left(|x-y|, |1+x-y|, |1+y-x|\right)^2, \quad x, y \in [0,1)$$

In this case, workers and firms are placed on a unit circle, with 0 indicating an arbitrary starting point of the circle and 1 indicating the end point of the circle (and therefore overlaps with 0). Agents are symmetric but not identical to each other: each worker produce the most when matched with a firm located closest to them on the unit circle, and produce less as the distance increases.

The production function is illustrated in Figure A-2.

Figure A-2: Production Function $f(x,y) = a - b\min\left(|x-y|, |1+x-y|, |1+y-x|\right)^2$



Notes: This graph illustrates the production function with horizontal heterogeneity, specified by the circular form. The left panel shows the function with respect to y at different levels of x. The right panel shows the function in 3-D format.

IV.2 Precautionary Mismatch vs. No Precautionary Motive

In this section we estimate the extent of precautionary mismatch (misallocation due to workers' precautionary motive) by conducting an accounting exercise, where all unemployed workers are assumed to search as if they are wealthy, i.e., they follow the search strategy of the wealthiest workers among their own productivity types.

Let all variables with superscript PM denote equilibrium objects under the baseline equilibrium with precautionary mismatch, and variables with superscript NP denote equilibrium objects under the hypothetical scenario where workers search like the wealthiest and thus behave as if they have no precautionary motive.

Let $X \in \{PM, NP\}$ denote the share of employed workers with productivity type x and working at jobs with productivity type y as $s^X(x,y)$. We have

$$s^X(x,y) = \frac{\int_a d_m(a,x,y)\,\mathrm{d}a}{\iiint d_m(a,x,y)\,\mathrm{d}a\,\mathrm{d}x\,\mathrm{d}y} \in [0, d_w(x)]\,.$$

Then the difference in aggregate labor productivity in the two economies can be expressed as

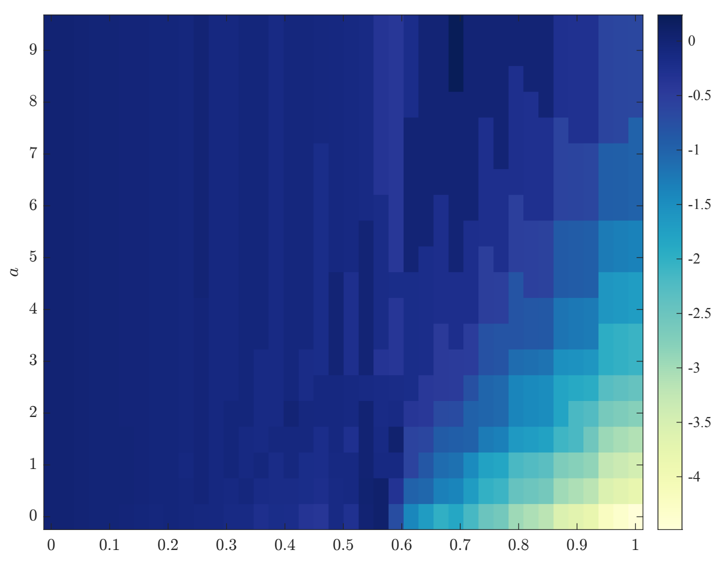
$$\iint f(x,y)\,s^{PM}(x,y)\,\mathrm{d}x\,\mathrm{d}y - \iint f(x,y)\,s^{NP}(x,y)\,\mathrm{d}x\,\mathrm{d}y.$$

We can also decompose the aggregate labor productivity difference by workers at different states. For workers with wealth holdings a and productivity type x in the baseline economy, their average difference in productivity compared with the no-precautionary-motive scenario is

$$\frac{\int f(x,y) d_m^{PM}(a,x,y)\,\mathrm{d}y}{\int d_m^{PM}(a,x,y)\,\mathrm{d}y} - \frac{\int f(x,y) s^{NP}(x,y)\,\mathrm{d}y}{\int s^{NP}(x,y)\,\mathrm{d}y}.$$

We plot the productivity difference in Figure A-3. As we can see, the productivity loss due to precautionary motive is concentrated on high-skilled wealth-poor workers, who tend to be young labor market entrants. The overall productivity in the NP economy is around 0.15% higher than the baseline level.

Figure A-3: Average Productivity Difference b/w PM and NP by a and x



Notes: This graph illustrates the average productivity differences of workers in different states between the baseline economy (PM) and the hypothetical case without precautionary motive (NP).