Models of Labor Market Monopsony

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Basic Monopsony Model

- Insight attributed to Robinson (1933)’s book *The Economics of Imperfect Competition*

- “Monopsony” literally means a market with a single buyer

- Profit maximizing firm takes into account upward-sloping labor supply curve

\[
\max_N R(N) - w(N)N
\]

- The first order condition is

\[
R'(N) = w(N) + w'(N)N \implies w(N) = \left[1 + \frac{1}{\varepsilon(N)}\right]^{-1} R'(N)
\]

where \(\varepsilon(N) := \frac{w(N)}{w'(N)N}\) is the labor supply elasticity

- Underemployment and underpay relative to the perfect competition benchmark
Sources of Monopsony Power

- The literal sole-employer case is rarely realistic (e.g., Méndez-Chacón and Van Patten, 2021)

- Oligopsony: e.g., Cournot model of employment-setting game

\[
\max_{n_i} R_i(n_i) - w\left(n_i + n_{-i}^*\right) n_i \Rightarrow w(N) = \left[1 + \frac{1}{\varepsilon(N)} \frac{n_i}{N}\right]^{-1} \frac{R'_i(n_i)}{\text{MRPL}}
\]

- Monopsonistic competition: atomistic firms face firm-specific labor supply curves

\[
\max_{n_i} R_i(n_i) - w_i(n_i) n_i \Rightarrow w_i(n_i) = \left[1 + \frac{1}{\varepsilon_i(n_i)}\right]^{-1} \frac{R'_i(n_i)}{\text{MRPL}}
\]

- What are the sources of \(w_i(n_i)\) being upward sloping?
Microfoundation 1: Search Friction

  - Unemployment rate: \( u \lambda = (1 - u) \delta \Rightarrow u = \frac{\delta}{\delta + \lambda} \)
  - Employed distribution: \( (1 - u) G(w; F)[\delta + \lambda(1 - F(w))] = u \lambda F(w) \Rightarrow G(w; F) = \frac{\delta F(w)}{\delta + \lambda[1-F(w)]} \)
  - Firm size: \( \{\delta + \lambda [1 - F(w)]\} n(w; F) = \frac{1}{M_f} [M_w u \lambda + M_w (1 - u) G(w; F) \lambda] \)  
    \[ \Rightarrow n(w; F) = \frac{M_w}{M_f} \frac{\delta \lambda}{[\delta + \lambda(1 - F(w))]^2} \]
  - Equilibrium offer distribution: \( \pi(w; F) = (p - w) n(w; F) \Rightarrow F(w) = \frac{\delta + \lambda}{\lambda} \left[1 - \sqrt{\frac{p-w}{p-b}}\right] \)

- Key insight: Search friction sustains wage dispersion among homogeneous workers and firms. A higher wage attracts more workers at the expense of a lower profit per worker.
- As \( \lambda \to \infty \), it goes to perfect competition
Microfoundation 2: Preference Idiosyncrasy

- IO style differentiated products perspective (Card et al., 2018; Lamadon et al., 2021)

- The indirect utility of worker $i$ working at firm $j$ is $u_{ij} = \ln w_j + \ln a_j + \epsilon_{ij}$

- Heterogeneous preference over workplace differentiation (location, corporate culture, etc.)

- $\epsilon_{ij}$ are i.i.d. draws from a type I extreme value distribution à la McFadden (1973)

$$
\Pr \left( \arg\max_{k\in\{1, \ldots, J\}} \{u_{ik}\} = j \right) = \frac{\exp \left[ \frac{(\ln w_j + \ln a_j)}{\sigma} \right]}{\sum_k \exp \left[ \frac{(\ln w_k + \ln a_k)}{\sigma} \right]} = \frac{(w_j a_j)^{\frac{1}{\sigma}}}{\sum_k (w_k a_k)^{\frac{1}{\sigma}}}
$$

- Assume $J$ is large so the firm-specific labor supply functions are $n_j (w_j) = N \lambda (w_j a_j)^{\frac{1}{\sigma}}$
  - As $\sigma \to 0$, it goes to perfect competition
  - As $\sigma \to \infty$, wage is useless in attracting employment

- Alternative: pose a Dixit-Stiglitz CES style preference structure (Berger et al., 2021)
Measurement of Monopsony Power

1. Concentration approach (Azar et al., 2020; Benmelech et al., 2020; Rinz, 2020)
   - Killed by modern empirical IO (abandons the structure-conduct-performance paradigm)
   - Relationship between concentration and market power depends on assumed market structure

2. Elasticity approach (several papers in 2010 JOLE Special Issue)
   - Estimate firm-specific labor supply elasticity (akin to demand estimation in IO)
   - Mostly for a specific market (Nurses, Teachers, etc.)
   - Variants: wage elasticity of separation, recruitment, or applications

3. Production approach (popularized by De Loecker et al., 2020, on the product market)
   - Markup as output elasticity (production estimation) divided by revenue share (observed) $\mu = \frac{\theta_V}{\alpha_V}$
   - Hershbein et al. (2021) extends the production approach to allow for labor market power

\[
1 + \frac{1}{\varepsilon_L} = \mu^{-1} \cdot \theta_L \cdot \alpha_L^{-1}
\]
Granularity vs. Atomicity

▶ Are firms granular or atomistic? (Do firms compete strategically?)

▶ Intuition of “size” as a source of market power?

▶ Recent developments

▶ Berger, Herkenhoff and Mongey (2021) develop an oligopsony model borrowed from Atkeson and Burstein (2008)’s nested CES preference structure

▶ Jarosch, Nimczik and Sorkin (2021) propose a random search and bargaining model where firm size affects worker’s outside option

▶ Roussille and Scuderi (2021) perform non-nested model comparison tests and argue models ignoring strategic interactions in wage setting outperform models with strategic interactions

▶ Focus on Berger, Herkenhoff and Mongey (2021) today. The market structure is given by

▶ Continuum of local labor markets $j \in [0, 1]$

▶ Each local labor market $j$ has an exogenous and finite number of firms $i \in \{1, 2, \ldots, m_j\}$
Representative Household

- **Preferences**

\[
\max_{\{n_{ijt}, c_{ijt}, K_{t+1}\}} \sum_{t=0}^{\infty} \beta^t U(C_t, N_t)
\]

- **Aggregate consumption**

\[
C_t := \int_0^1 \sum_{i \in j} c_{ijt} \, dj
\]

- **Disutility of labor supply**

\[
N_t := \left[ \int_0^1 n_{jt}^{\frac{\theta+1}{\theta}} \, dj \right]^{\frac{\theta}{\theta+1}}, \quad n_{jt} := \left[ \sum_{i \in j} n_{ijt}^\eta \right]^{\frac{\eta}{\eta+1}}, \quad \eta > \theta
\]

- **Budget constraint**

\[
C_t + [K_{t+1} - (1 - \delta)K_t] = \int_0^1 \sum_{i \in j} w_{ijt} n_{ijt} \, dj + R_t K_t + \Pi_t
\]
Representative Household

Define market and aggregate wage index such that \( w_{jt} n_{jt} = \sum_{i \in j} w_{ijt} n_{ijt} \), \( W_{t} N_{t} = \int_{0}^{1} w_{jt} n_{jt} \, dj \).

1. Aggregate-level labor supply
\[
W_{t} = - \frac{U_{N}(C_{t}, N_{t})}{U_{C}(C_{t}, N_{t})}
\]

2. Market-level labor supply
\[
n_{jt} = \left( \frac{w_{jt}}{W_{t}} \right)^{\theta} N_{t} \Leftrightarrow w_{jt} = \left( \frac{n_{jt}}{N_{t}} \right)^{\frac{1}{\theta}} W_{t}
\]

3. Firm-level labor supply
\[
n_{ijt} = \left( \frac{w_{ijt}}{w_{jt}} \right)^{\eta} \left( \frac{w_{jt}}{W_{t}} \right)^{\theta} N_{t} \Leftrightarrow w_{ijt} = \left( \frac{n_{ijt}}{n_{jt}} \right)^{\frac{1}{\eta}} \left( \frac{n_{jt}}{N_{t}} \right)^{\frac{1}{\theta}} W_{t}
\]

4. Wage index
\[
W_{t} = \left[ \int_{0}^{1} w_{jt}^{1+\theta} \, dj \right]^{\frac{1}{1+\theta}}, \quad w_{jt} = \left[ \sum_{i \in j} w_{ijt}^{1+\eta} \right]^{\frac{1}{1+\eta}}
\]

Derivation

Microfoundation
Discussion on the Elasticities

- **Across-market elasticity of substitution \( \theta \)**
  - As \( \theta \to 0 \): equal division of workers across markets \( n_{jt} = n_{j't} \) regardless of wage index
  - As \( \theta \to \infty \): send all workers to the market with the highest wage index

- **Within-market, across-firm elasticity of substitution \( \eta \)**
  - As \( \eta \to 0 \): equal division of workers across firms \( n_{ijt} = n_{i'jt} \) regardless of wage
  - As \( \eta \to \infty \): send all workers to the firm with the highest wage (competitive local labor markets)

- **Two limiting cases of monopsonistic competition**
  1. \( \theta \to \eta \)
     \[
     N_t = \left[ \int_0^1 \left( \left( \sum_{i \in j} n_{ijt}^{\frac{\eta+1}{\eta}} \right)^{\frac{\theta+1}{\theta}} \right) dj \right]^{\frac{\theta}{\theta+1}} \to \left[ \int_0^1 \sum_{i \in j} n_{ijt}^{\frac{\eta+1}{\eta}} dj \right]^{\frac{\eta}{\eta+1}}
     \]
  2. \( m_j \to \infty \)
     \[
     N_t \to \left[ \int_0^1 \left( \left( \int_{i \in j} n_{ijt}^{\frac{\eta+1}{\eta}} di \right)^{\frac{\theta+1}{\theta}} \right) dj \right]^{\frac{\theta}{\theta+1}} \text{ with symmetry } \to \left[ \int_{i \in j} n_{ijt}^{\frac{\eta+1}{\eta}} di \right]^{\frac{\eta}{\eta+1}}
     \]
Heterogeneous Firms

- Atomistic wrt the macroeconomy, so firms take economy-wide aggregates \( W_t, N_t \) as given

- Granular wrt a local labor market, so firms are Cournot competing within a market

\[
\max_{n_{ijt}, k_{ijt}} z_{ijt} \left( k_{ijt}^{1-\gamma} n_{ijt}^{\gamma} \right)^{\alpha} - R_t k_{ijt} - w \left( n_{ijt}, n_{-ijt}^*, W_t, N_t \right) n_{ijt}
\]

\( z_{ijt} n_{ijt}^{\tilde{\alpha}} \)

- I.e., firms take as given their local competitors’ employment decisions \( n_{-ijt}^* \), but do internalize the effect of their own decision \( n_{ijt} \) on the market-level aggregate \( n_j \)

\[
w \left( n_{ijt}, n_{-ijt}^*, W_t, N_t \right) = \left( \frac{n_{ijt}}{n_j} \right)^{\frac{1}{\eta}} \left( \frac{n_j}{N_t} \right)^{\frac{1}{\theta}} W_t, \quad n_j = \left[ \frac{n_{ijt}^{\frac{\eta+1}{\eta}}}{n_{ijt}^{\frac{\eta}{\eta}}} + \sum_{k \neq i, k \in j} n_{kjt}^{\frac{\eta+1}{\eta}} \right]^{\frac{\eta}{\eta+1}}
\]

- The (inverse) firm-specific labor supply elasticity is

\[
\frac{\partial \log w \left( n_{ijt}, n_{-ijt}^*, W_t, N_t \right)}{\partial \log n_{ijt}} = \frac{1}{\eta} + \left( \frac{1}{\theta} - \frac{1}{\eta} \right) \frac{s_{ijt}}{w_{ijt} n_{ijt}} = s_{ijt} \frac{1}{\theta} + \left( 1 - s_{ijt} \right) \frac{1}{\eta}
\]
Partial Equilibrium

\[
\begin{align*}
    w_{ijt}^* &= \left[1 + \frac{1}{\varepsilon_{ijt}^*}\right]^{-1} \quad \text{MRPL}_{ijt}, \\
    \varepsilon_{ijt}^* &= \left[s_{ijt} \frac{1}{\theta} + (1 - s_{ijt}) \frac{1}{\eta}\right]^{-1}
\end{align*}
\]

A. Low productivity firm

B. High productivity firm

Figure from Berger, Herkenhoff and Mongey (2021)
Market Equilibrium

A. Share of market payroll: $s_{ij}$

B. Markup: $\mu_{ij} = \frac{\varepsilon(s_{ij})}{\varepsilon(s_{ij}) + 1}$

C. Wage ($\times 1000$): $w_{ij} = \mu_{ij} m r p_{ij}$

D. Employment: $n_{ij}$

Figure from Berger, Herkenhoff and Mongey (2021)
General Equilibrium

Aggregate markdown $\mu$ and misallocation $\omega$

$$W = \mu \tilde{\alpha} \tilde{Z} N^{\tilde{\alpha} - 1}, \quad \tilde{Y} = \omega \tilde{Z} N^{\tilde{\alpha}}$$

Benchmark cases

1. Efficient allocation ($w_{ijt} = \text{MRPL}_{ijt}$ firm by firm): $\mu = 1$ and $\omega = 1$
2. Monopsonistic competition limits ($\theta \to \eta$ or $m_j \to \infty$): $\mu = \mathbb{E} [\mu_{ij}] = \eta / (\eta + 1)$ and $\omega = 1$
3. BHM oligopsonistic economy: $\mu < \mathbb{E} [\mu_{ij}]$ and $\omega < 1$

Labor share and concentration

$$LS = \alpha \gamma \times \left\{ \text{competitive LS} \times \left[ HHI^{wn} \left( \frac{\theta}{\theta + 1} \right)^{-1} + (1 - HHI^{wn}) \left( \frac{\eta}{\eta + 1} \right)^{-1} \right]^{-1} \right\}$$

(BHM find wage-bill HHI actually declined during the past 30 years)
Identification

Structural elasticities, if known, would identify \((\theta, \eta)\)

\[
\varepsilon (s_{ij}; \theta, \eta) := \left. \frac{\partial \log n_{ij}}{\partial \log w_{ij}} \right|_{n^*_{-ij}} = \left[ s_{ij} \frac{1}{\theta} + (1 - s_{ij}) \frac{1}{\eta} \right]^{-1}
\]

Reduced form elasticities are what we observe, even with a perfect instrument

\[
\varepsilon (s_{ij}, \theta, \eta, \ldots) := \frac{d \log n_{ij}}{d \log w_{ij}} \approx \frac{\varepsilon (s_{ijt}; \theta, \eta)}{1 + \varepsilon (s_{ijt}, \theta, \eta) \left( \frac{\eta - \theta}{\theta \eta} \right) \left( \sum_{k \neq i} s_{kjt} \frac{d \log n_{kjt}}{d \log n_{ijt}} \right)}
\]

Under monopsonistic competition limits \((\theta \to \eta \text{ or } m_j \to \infty)\): \(\varepsilon = \varepsilon\)

Potential biases

- Positive idiosyncratic productivity shock to firm \(i\): \(d \log n_{kjt} < 0 \Rightarrow \epsilon > \varepsilon\)
- Non-idiosyncratic positive shock common across firms: \(d \log n_{kjt} > 0 \Rightarrow \epsilon < \varepsilon\)
Empirical Evidence

- Estimation: size-dependent reduced-form labor supply elasticities
  - State corporate tax changes as labor demand shocks (Giroud and Rauh, 2019)
  - Indirect inference: $\min_{\theta, \eta} |\hat{\epsilon}_{\text{Data}}(s) - \hat{\epsilon}_{\text{Model}}(s, \theta, \eta)|$

- Validation
  - Incomplete pass-through of value added to wages (Kline et al., 2019)
    \[ \Delta \log w_{ijt} = \Delta \log \mu_{ijt} + \Delta \log vapw_{ijt} \]
  - Responses of firms to competitors’ wage changes (Staiger et al., 2010)
    \[ \Delta \log w_{ij} = \Omega(s_{ij}) \Delta \log vapw_{ij} + (1 - \Omega(s_{ij})) \sum_{k \neq i} \left( \frac{s_{kj}}{1 - s_{ij}} \right) \Delta \log w_{kj} \]
    where $\Omega(s_{ij}) = \frac{s_{ij}(\eta - \theta) + \theta(\eta + 1)}{[1 + (1 + \eta)(1 - s_{ij})]s_{ij}(\eta - \theta) + \theta(\eta + 1)}$
  - Employment and wage effects of mergers (Arnold, 2021)
Jarosch, Nimczik and Sorkin (2021)

- Wage bargaining, not monopsonistic wage setting
- Relate firms’ bargaining leverage to firm “size” $f_i$ (here firm “size” is reduced form)
- In a standard model, worker’s outside option in bargaining is

$$U = b + \beta \left( \lambda \sum_i f_i W_i + (1 - \lambda)U \right)$$

- Here firm can remove itself from worker’s outside option (outside options are truly outside)

$$U_i = b + \beta \left( \lambda \sum_{j \neq i} f_j W_j + \lambda f_i W_i + (1 - \lambda (1 - f_i) - \lambda f_i) U_i \right)$$

where $\lambda$ is the probability that the worker is the only applicant

- Bargaining happens only within a single worker-firm pair
Conclusion

- Recent resurgent interests in labor market power
  - Many job market papers on this topic as a sign
  - Speculations on candidate explanation for falling labor share and rising inequality
  - Worries often heard about mega-firms, demise of unions, anti-competitive labor contracts

- Monopsonistic wage setting or bargaining?
  - Monopsonistic view: markdown (deviation of $w$ from MRPL) measures labor market power
  - Is $w = MRPL$ the most relevant benchmark? If search friction is technological, it does not correspond to the (constrained) efficient benchmark in DMP (Hosios condition).
  - Both are empirically challenging
    - Monopsonistic view: MRPL, hence markdown, is intrinsically unobserved
    - Bargaining view: bargaining power and outside option are difficult to measure

- IO has developed many tools to study market power and strategic interactions during past decades. It might be fruitful for labor and IO economists to talk to each other.
References I


References II


Appendix
This Year’s Job Market Papers on Labor Market Power

- Iain Bamford (Columbia), “Monopsony Power, Spatial Equilibrium, and Minimum Wages”
- Nikhil Datta (UCL), “Local Monopsony Power”
- Mayara Felix (MIT), “Trade, Labor Market Concentration, and Wages”
- Benjamin Scuderi (Berkeley), “Bidding for Talent: Equilibrium Wage Dispersion on a High-Wage Online Job Board”
- ...
The intra-temporal Euler equation (i.e., FOCs for consumption and labor supply to firm-ij)

\[ wi_{jt} = \frac{\partial n_{jt}}{\partial n_{ijt}} \frac{\partial N_t}{\partial n_{jt}} \left( -\frac{U_N(C_t, N_t)}{U_C(C_t, N_t)} \right) \]

The labor supply disutility functional form delivers

\[ \frac{\partial N_t}{\partial n_{jt}} \frac{n_{jt}}{N_t} = \left( \frac{n_{jt}}{N_t} \right)^{\frac{\theta+1}{\theta}}, \quad \frac{\partial n_{jt}}{\partial n_{ijt}} \frac{n_{ijt}}{n_{jt}} = \left( \frac{n_{ijt}}{n_{jt}} \right)^{\frac{\eta+1}{\eta}} \]

Therefore

\[ \int_{0}^{1} \frac{\partial N_t}{\partial n_{jt}} \frac{n_{jt}}{N_t} dj = N_t \frac{\theta+1}{\theta} \int_{0}^{1} n_{jt}^{\frac{\theta+1}{\theta}} dj = 1, \quad \sum_{i \in j} \frac{\partial n_{jt}}{\partial n_{ijt}} \frac{n_{ijt}}{n_{jt}} = n_{jt}^{-\frac{\eta+1}{\eta}} \sum_{i \in j} n_{ijt}^{\frac{\eta+1}{\eta}} = 1. \]
Derivation for Household Problem II

Using these equations we have

\[ W_t N_t = \int_0^1 \sum_{i \in j} \left( \frac{\partial n_{jt}}{\partial n_{ijt}} \right) \left( \frac{\partial N_t}{\partial n_{jt}} \right) \left( -\frac{U_N(C_t, N_t)}{U_C(C_t, N_t)} \right) N_t \, dj = \left( -\frac{U_N(C_t, N_t)}{U_C(C_t, N_t)} \right) N_t \]

Therefore, we have the aggregate labor supply curve

\[ W_t = -\frac{U_N(C_t, N_t)}{U_C(C_t, N_t)} \]

The market labor supply curve can be obtained by

\[ w_{jt} n_{jt} = \sum_{i \in j} w_{ijt} n_{ijt} = \left( \frac{n_{jt}}{N_t} \right)^{\frac{\theta + 1}{\theta}} W_t N_t \Rightarrow n_{jt} = \left( \frac{w_{jt}}{W_t} \right)^{\theta} N_t \]

Lastly, the firm-specific labor supply curve

\[ w_{ijt} n_{ijt} = \left( \frac{n_{ijt}}{n_{jt}} \right)^{\frac{\eta + 1}{\eta}} \left( \frac{w_{jt} n_{jt}}{W_t N_t} \right) W_t N_t \Rightarrow n_{ijt} = \left( \frac{w_{ijt}}{w_{jt}} \right)^{\eta} n_{jt} \]
Microfoundation: Logit and CES

- Worker $l$’s disutility of working $h_{lij}$ hours at firm $ij$ are:
  
  \[ v_{lij} = e^{-\xi_{lij}} h_{lij}, \quad \log v_{lij} = \log h_{ij} - \xi_{lij} \]

- The random utility term $\xi_{lij}$ is distributed iid from a multi-variate Gumbel distribution

  \[ F(\xi_{i1}, \ldots, \xi_{NJ}) = \exp \left[ -\sum_{ij} e^{-(1+\eta)\xi_{ij}} \right] \]

- Suppose each worker must earn $y_l \sim G(y)$ such that $h_{lij}$ satisfies $y_l = w_{ij} h_{lij}$. Worker $l$ solves

  \[ \min_{ij} \{ \log h_{ij} - \xi_{lij} \} \equiv \max_{ij} \{ \log w_{ij} - \log y_l + \xi_{lij} \} \]

- This problem delivers the following choice probability for worker $l$

  \[ \Pr(w_{ij}, w_{-ij}; l) = \frac{w_{ij}^{1+\eta}}{\sum_{ij} w_{ij}^{1+\eta}} \]
Microfoundation: Logit and CES

- Total labor supply to firm $ij$ is

$$n_{ij} = \int \Pr (w_{ij}, w_{-ij}; l) \times h_{ij} dG (y_l) = \frac{w_{ij}^{\eta}}{\sum_{ij} w_{ij}^{1+\eta}} \int y_l G (y_l)$$

- Aggregating this expression we obtain the obvious result that $\sum_{ij} w_{ij} n_{ij} = Y$

- Define indexes $W := \left[ \sum_{ij} w_{ij}^{1+\eta} \right]^{\frac{1}{1+\eta}}$, $N := \left[ \sum_{ij} n_{ij}^{\eta} \right]^{\frac{\eta+1}{\eta}}$. It holds that $WN = Y$.

- Thus

$$n_{ij} = \frac{w_{ij}^{\eta}}{\sum_{ij} w_{ij}^{1+\eta}} Y = \frac{w_{ij}^{\eta}}{W^{1+\eta}} WN = \left( \frac{w_{ij}}{W} \right)^{\eta} N$$
Microfoundation: Nested Logit and Nested CES

- Preference shock distribution \( F(\xi_{i1}, \ldots, \xi_{NJ}) = \exp \left[ - \sum_{j=1}^{J} \left( \sum_{i=1}^{M_j} e^{-(1+\eta)\xi_{ij}} \right)^{\frac{1+\theta}{1+\eta}} \right] \)

- The choice probabilities can be expressed as

\[
\Pr(w_{ij}, w_{-ij}) = \frac{w_{ij}^{1+\eta}}{\sum_{i=1}^{M_j} w_{ij}^{1+\eta}} \times \frac{\left[ \sum_{i=1}^{M_j} w_{ij}^{1+\eta} \right]^{\frac{1+\theta}{1+\eta}}}{\sum_{l=1}^{J} \left[ \sum_{k=1}^{M_l} w_{kl}^{1+\eta} \right]^{\frac{1+\theta}{1+\eta}}}
\]

Pr(Choose firm \( i \) | Choose market \( j \))

- Following the same steps as before

\[
n_{ij} = \frac{w_{ij}^{\eta}}{\sum_{i=1}^{M_j} w_{ij}^{1+\eta}} \times \frac{\left[ \sum_{i=1}^{M_j} w_{ij}^{1+\eta} \right]^{\frac{1+\theta}{1+\eta}}}{\sum_{l=1}^{J} \left[ \sum_{k=1}^{M_l} w_{kl}^{1+\eta} \right]^{\frac{1+\theta}{1+\eta}}}
\]

- See Dupuy and Galichon (2014) for the formalism of the continuum limit