Models of Labor Market Monopsony

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Basic Monopsony Model

- Insight attributed to Robinson (1933)'s book *The Economics of Imperfect Competition*
- "Monopsony" literally means a market with a single buyer
- Profit maximizing firm takes into account upward-sloping labor supply curve

$$\max_{N} \; R(N) - w(N)\,N$$

- The first order condition is

$$\underbrace{R'(N)}_{\text{MRPL}} = \underbrace{w(N) + w'(N)N}_{\text{marginal cost of labor}} \implies \underbrace{w(N)}_{\text{wage}} = \underbrace{\left[1 + \frac{1}{\varepsilon(N)}\right]^{-1}}_{\text{markdown}} \underbrace{R'(N)}_{\text{MRPL}}$$

where $\varepsilon(N) := \frac{w(N)}{w'(N)N}$ is the labor supply elasticity

- Underemployment and underpay relative to the perfect competition benchmark

Sources of Monopsony Power

- The literal sole-employer case is rarely realistic (e.g., Méndez-Chacón and Van Patten, 2021)
- Oligopsony: e.g., Cournot model of employment-setting game

$$\max_{n_i} R_i(n_i) - w\left(n_i + n^*_{-i}\right) n_i \implies \underbrace{w(N)}_{\text{wage}} = \underbrace{\left[1 + \frac{1}{\varepsilon(N)} \frac{n_i}{N}\right]^{-1}}_{\text{markdown}} \underbrace{R_i'(n_i)}_{\text{MRPL}}$$

- Monopsonistic competition: atomistic firms face firm-specific labor supply curves

$$\max_{n_i} \; R_i(n_i) - w_i(n_i) n_i \implies \underbrace{w_i(n_i)}_{\text{wage}} = \underbrace{\left[1 + \frac{1}{\varepsilon_i(n_i)}\right]^{-1}}_{\text{markdown}} \underbrace{R_i'(n_i)}_{\text{MRPL}}$$

- What are the sources of $w_i(n_i)$ being upward sloping?

Microfoundation 1: Search Friction

- Manning (2003) based on Burdett and Mortensen (1998). In steady state:
  - Unemployment rate: $u\lambda = (1-u)\delta \Rightarrow u = \frac{\delta}{\delta+\lambda}$
  - Employed distribution: $(1-u)G(w;F)[\delta + \lambda(1-F(w))] = u\lambda F(w) \Rightarrow G(w;F) = \frac{\delta F(w)}{\delta+\lambda[1-F(w)]}$
  - Firm size: $\underbrace{\{\delta + \lambda\,[1-F(w)]\}}_{\text{separation rate}} n(w;F) = \underbrace{\frac{1}{M_f}[M_w u\lambda + M_w(1-u)G(w;F)\lambda]}_{\text{recruitment}}$

$$\Rightarrow n(w;F) = \frac{M_w}{M_f}\frac{\delta\lambda}{[\delta+\lambda(1-F(w))]^2}$$

  - Equilibrium offer distribution: $\pi(w;F) = (p-w)n(w;F) \Rightarrow F(w) = \frac{\delta+\lambda}{\lambda}\left[1-\sqrt{\frac{p-w}{p-b}}\right]$
- Key insight: Search friction sustains wage dispersion among homogeneous workers and firms. A higher wage attracts more workers at the expense of a lower profit per worker.
- As $\lambda \to \infty$, it goes to perfect competition (all employed at $w = p$)

Microfoundation 2: Preference Idiosyncrasy

- IO style differentiated products perspective (Card et al., 2018; Lamadon et al., 2021)
- The indirect utility of worker i working at firm j is $u_{ij} = \ln w_j + \ln a_j + \epsilon_{ij}$
- Heterogeneous preference over workplace differentiation (location, corporate culture, etc.)
- ϵ_{ij} are i.i.d. draws from a type I extreme value distribution à la McFadden (1973)

$$\Pr\left(\arg\max_{k\in\{1,\dots,J\}} \{u_{ik}\} = j\right) = \frac{\exp\left[\left(\ln w_j + \ln a_j\right)/\sigma\right]}{\sum_k \exp\left[\left(\ln w_k + \ln a_k\right)/\sigma\right]} = \frac{\left(w_j a_j\right)^{\frac{1}{\sigma}}}{\sum_k \left(w_k a_k\right)^{\frac{1}{\sigma}}}$$

- Assume J is large so the firm-specific labor supply functions are $n_j\left(w_j\right) = \mathcal{N}\lambda\left(w_j a_j\right)^{\frac{1}{\sigma}}$
  - As $\sigma \to 0$, it goes to perfect competition
  - As $\sigma \to \infty$, wage is useless in attracting employment
- Alternative: pose a Dixit-Stiglitz CES style preference structure (Berger et al., 2021)

Measurement of Monopsony Power

1. Concentration approach (Azar et al., 2020; Benmelech et al., 2020; Rinz, 2020)
   - Killed by modern empirical IO (abandons the structure-conduct-performance paradigm)
   - Relationship between concentration and market power depends on assumed market structure
2. Elasticity approach (several papers in 2010 JOLE Special Issue)
   - Estimate firm-specific labor supply elasticity (akin to demand estimation in IO)
   - Mostly for a specific market (Nurses, Teachers, etc.)
   - Variants: wage elasticity of separation, recruitment, or applications
3. Production approach (popularized by De Loecker et al., 2020, on the product market)
   - Markup as output elasticity (production estimation) divided by revenue share (observed) $\mu = \frac{\theta_V}{\alpha_V}$
   - Hershbein et al. (2021) extends the production approach to allow for labor market power

$$\underbrace{1 + \frac{1}{\varepsilon_L}}_{\text{(inverse) markdown}} = \underbrace{\mu^{-1}}_{\text{markup}} \cdot \underbrace{\theta_L}_{\text{output elasticity}} \cdot \underbrace{\alpha_L^{-1}}_{\text{labor share}}$$

Granularity vs. Atomicity

- Are firms granular or atomistic? Do firms compete strategically?
- Intuition of "size" as a source of market power?
- Recent developments
  - Berger, Herkenhoff and Mongey (2021) develop an oligopsony model borrowed from Atkeson and Burstein (2008)'s nested CES preference structure
  - Jarosch, Nimczik and Sorkin (2021) propose a random search and bargaining model where firm size affects worker's outside option
  - Roussille and Scuderi (2021) perform non-nested model comparison tests and argue models ignoring strategic interactions in wage setting outperform models with strategic interactions
- Focus on Berger, Herkenhoff and Mongey (2021) today. The market structure is given by
  - Continuum of local labor markets $j \in [0, 1]$
  - Each local labor market j has an exogenous and finite number of firms $i \in \{1, 2, \ldots, m_j\}$

Representative Household

- Preferences

$$\max_{\{n_{ijt}, c_{ijt}, K_{t+1}\}_{t=0}^{\infty}} \sum_{t=0}^{\infty} \beta^t U(\mathbf{C}_t, \mathbf{N}_t)$$

- Aggregate consumption

$$\mathbf{C}_t := \int_0^1 \sum_{i \in j} c_{ijt} \mathrm{d}j$$

- Disutility of labor supply

$$\mathbf{N}_t := \left[\int_0^1 \mathbf{n}_{jt}^{\frac{\theta+1}{\theta}} \mathrm{d}j \right]^{\frac{\theta}{\theta+1}}, \quad \mathbf{n}_{jt} := \left[\sum_{i \in j} n_{ijt}^{\frac{\eta+1}{\eta}} \right]^{\frac{\eta}{\eta+1}}, \quad \eta > \theta$$

- Budget constraint

$$\mathbf{C}_t + [K_{t+1} - (1-\delta)K_t] = \int_0^1 \sum_{i \in j} w_{ijt} n_{ijt} dj + R_t K_t + \Pi_t$$

Representative Household

Define market and aggregate wage index such that $\mathbf{w}_{jt}\mathbf{n}_{jt} = \sum_{i\in j} w_{ijt} n_{ijt}$, $\mathbf{W}_t\mathbf{N}_t = \int_0^1 \mathbf{w}_{jt}\mathbf{n}_{jt} dj$.

1. Aggregate-level labor supply

$$\mathbf{W}_t = -\frac{U_N(\mathbf{C}_t, \mathbf{N}_t)}{U_C(\mathbf{C}_t, \mathbf{N}_t)}$$

2. Market-level labor supply

$$\mathbf{n}_{jt} = \left(\frac{\mathbf{w}_{jt}}{\mathbf{W}_t}\right)^{\theta} \mathbf{N}_t \Leftrightarrow \mathbf{w}_{jt} = \left(\frac{\mathbf{n}_{jt}}{\mathbf{N}_t}\right)^{\frac{1}{\theta}} \mathbf{W}_t$$

3. Firm-level labor supply

$$n_{ijt} = \left(\frac{w_{ijt}}{\mathbf{w}_{jt}}\right)^{\eta} \left(\frac{\mathbf{w}_{jt}}{\mathbf{W}_t}\right)^{\theta} \mathbf{N}_t \Leftrightarrow w_{ijt} = \left(\frac{n_{ijt}}{\mathbf{n}_{jt}}\right)^{\frac{1}{\eta}} \left(\frac{\mathbf{n}_{jt}}{\mathbf{N}_t}\right)^{\frac{1}{\theta}} \mathbf{W}_t$$

4. Wage index

$$\mathbf{W}_t = \left[\int_0^1 \mathbf{w}_{jt}^{1+\theta} \mathrm{d}j\right]^{\frac{1}{1+\theta}}, \quad \mathbf{w}_{jt} = \left[\sum_{i\in j} w_{ijt}^{1+\eta}\right]^{\frac{1}{1+\eta}}$$

▸ Derivation ▸ Microfoundation

Discussion on the Elasticities

- Across-market elasticity of substitution θ
  - As $\theta \to 0$: equal division of workers across markets $\mathbf{n}_{jt} = \mathbf{n}_{j't}$ regardless of wage index
  - As $\theta \to \infty$: send all workers to the market with the highest wage index
- Within-market, across-firm elasticity of substitution η
  - As $\eta \to 0$: equal division of workers across firms $n_{ijt} = n_{i'jt}$ regardless of wage
  - As $\eta \to \infty$: send all workers to the firm with the highest wage (competitive local labor markets)
- Two limiting cases of monopsonistic competition

1. $\theta \to \eta$

$$\mathbf{N}_t = \left[\int_0^1 \left[\left(\sum_{i \in j} n_{ijt}^{\frac{\eta+1}{\eta}} \right)^{\frac{\eta}{\eta+1}} \right]^{\frac{\theta+1}{\theta}} \mathrm{d}j \right]^{\frac{\theta}{\theta+1}} \to \left[\int_0^1 \sum_{i \in j} n_{ijt}^{\frac{\eta+1}{\eta}} \mathrm{d}j \right]^{\frac{\eta}{\eta+1}}$$

2. $m_j \to \infty$

$$\mathbf{N}_t \to \left[\int_0^1 \left[\left(\int_{i \in j} n_{ijt}^{\frac{\eta+1}{\eta}} \mathrm{d}i \right)^{\frac{\eta}{\eta+1}} \right]^{\frac{\theta+1}{\theta}} \mathrm{d}j \right]^{\frac{\theta}{\theta+1}} \overset{\text{with symmetry}}{=} \left[\int_{i \in j} n_{ijt}^{\frac{\eta+1}{\eta}} \mathrm{d}i \right]^{\frac{\eta}{\eta+1}}$$

Heterogeneous Firms

- Atomistic wrt the macroeconomy, so firms take economy-wide aggregates $\mathbf{W}_t, \mathbf{N}_t$ as given
- Granular wrt a local labor market, so firms are Cournot competing within a market

$$\max_{n_{ijt}, k_{ijt}} \underbrace{z_{ijt}\left(k_{ijt}^{1-\gamma} n_{ijt}^{\gamma}\right)^{\alpha} - R_t k_{ijt}}_{\tilde{z}_{ijt} n_{ijt}^{\tilde{\alpha}}} - w\left(n_{ijt}, n_{-ijt}^{*}, \mathbf{W}_t, \mathbf{N}_t\right) n_{ijt}$$

- I.e., firms take as given their local competitors' employment decisions n_{-ijt}^{*}, but do internalize the effect of their own decision n_{ijt} on the market-level aggregate $\mathbf{n}_{jt}$

$$w\left(n_{ijt}, n_{-ijt}^{*}, \mathbf{W}_t, \mathbf{N}_t\right) = \left(\frac{n_{ijt}}{\mathbf{n}_{jt}}\right)^{\frac{1}{\eta}} \left(\frac{\mathbf{n}_{jt}}{\mathbf{N}_t}\right)^{\frac{1}{\theta}} \mathbf{W}_t, \quad \mathbf{n}_{jt} = \left[n_{ijt}^{\frac{\eta+1}{\eta}} + \sum_{k \neq i, k \in j} n_{kjt}^{*\,\frac{\eta+1}{\eta}}\right]^{\frac{\eta}{\eta+1}}$$

- The (inverse) firm-specific labor supply elasticity is

$$\frac{\partial \log w\left(n_{ijt}, n_{-ijt}^{*}, \mathbf{W}_t, \mathbf{N}_t\right)}{\partial \log n_{ijt}} = \frac{1}{\eta} + \left(\frac{1}{\theta} - \frac{1}{\eta}\right) \frac{\overbrace{w_{ijt} n_{ijt}}^{s_{ijt}}}{\mathbf{w}_{jt} \mathbf{n}_{jt}} = s_{ijt}\frac{1}{\theta} + (1 - s_{ijt})\frac{1}{\eta}$$

Partial Equilibrium

$$w^*_{ijt} = \underbrace{\left[1 + \frac{1}{\varepsilon^*_{ijt}}\right]^{-1}}_{\text{markdown}} \mathrm{MRPL}_{ijt}, \quad \varepsilon^*_{ijt} = \left[s_{ijt}\frac{1}{\theta} + (1 - s_{ijt})\frac{1}{\eta}\right]^{-1}$$

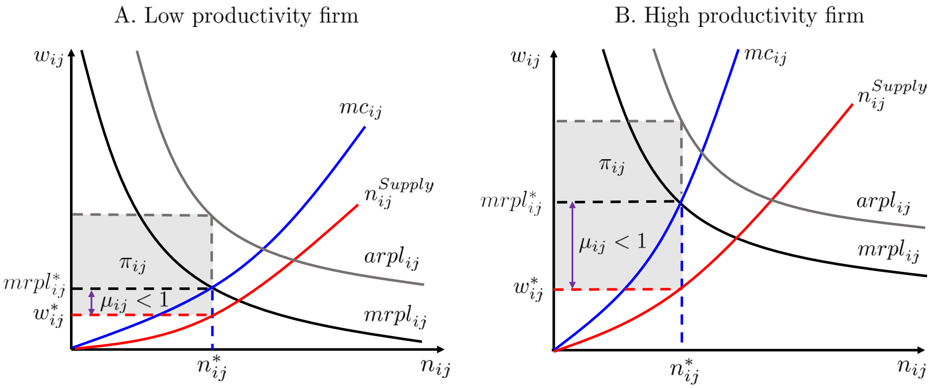

Figure from Berger, Herkenhoff and Mongey (2021)

Market Equilibrium

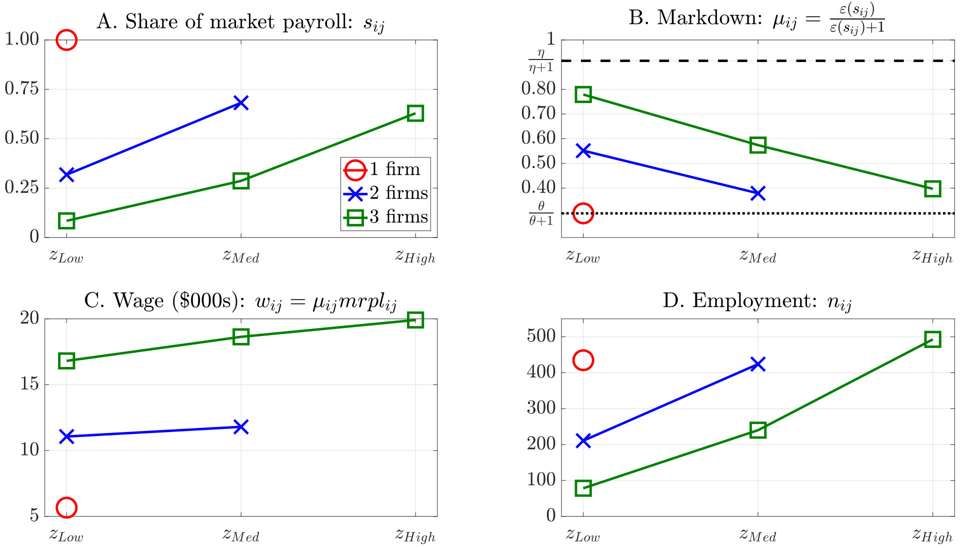

Figure from Berger, Herkenhoff and Mongey (2021)

General Equilibrium

- Aggregate markdown $\boldsymbol{\mu}$ and misallocation $\boldsymbol{\omega}$

$$\mathbf{W} = \boldsymbol{\mu}\tilde{\alpha}\tilde{\mathbf{Z}}\mathbf{N}^{\tilde{\alpha}-1}, \quad \tilde{\mathbf{Y}} = \boldsymbol{\omega}\tilde{\mathbf{Z}}\mathbf{N}^{\tilde{\alpha}}$$

- Benchmark cases

  1. Efficient allocation ($w_{ijt} = \text{MRPL}_{ijt}$ firm by firm): $\boldsymbol{\mu} = 1$ and $\boldsymbol{\omega} = 1$
  2. Monopsonistic competition limits ($\theta \to \eta$ or $m_j \to \infty$): $\boldsymbol{\mu} = \mathbb{E}[\mu_{ij}] = \eta/(\eta+1)$ and $\boldsymbol{\omega} = 1$
  3. BHM oligopsonistic economy: $\boldsymbol{\mu} < \mathbb{E}[\mu_{ij}]$ and $\boldsymbol{\omega} < 1$

- Labor share and concentration

$$LS = \underbrace{\alpha\gamma}_{\text{competitive LS}} \times \underbrace{\left[HHI^{wn}\left(\frac{\theta}{\theta+1}\right)^{-1} + (1 - HHI^{wn})\left(\frac{\eta}{\eta+1}\right)^{-1}\right]^{-1}}_{\text{labor market power adjustment}}$$

(BHM find wage-bill HHI actually declined during the past 30 years)

Identification

- Structural elasticities, if known, would identify (θ, η)

$$\varepsilon\left(s_{ij};\theta,\eta\right) := \left.\frac{\partial \log n_{ij}}{\partial \log w_{ij}}\right|_{n^*_{-ij}} = \left[s_{ij}\frac{1}{\theta} + \left(1-s_{ij}\right)\frac{1}{\eta}\right]^{-1}$$

- Reduced form elasticities are what we observe, even with a perfect instrument

$$\epsilon\left(s_{ij},\theta,\eta,\ldots\right) := \frac{\mathrm{d}\log n_{ijt}}{\mathrm{d}\log w_{ijt}} \approx \frac{\varepsilon\left(s_{ijt};\theta,\eta\right)}{1+\varepsilon\left(s_{ijt},\theta,\eta\right)\left(\frac{\eta-\theta}{\theta\eta}\right)\left(\sum_{k\neq i} s_{kjt}\frac{\mathrm{d}\log n_{kjt}}{\mathrm{d}\log n_{ijt}}\right)}$$

- Under monopsonistic competition limits ($\theta \to \eta$ or $m_j \to \infty$): $\epsilon = \varepsilon$
- Potential biases
  - Positive idiosyncratic productivity shock to firm i: $\mathrm{d}\log n_{kjt} < 0 \Rightarrow \epsilon > \varepsilon$
  - Non-idiosyncratic positive shock common across firms: $\mathrm{d}\log n_{kjt} > 0 \Rightarrow \epsilon < \varepsilon$

Empirical Evidence

- Estimation: size-dependent reduced-form labor supply elasticities
  - State corporate tax changes as labor demand shocks (Giroud and Rauh, 2019)
  - Indirect inference: $\min_{\theta,\eta} \left|\widehat{\epsilon}^{\text{Data}}(s) - \widehat{\epsilon}^{\text{Model}}(s, \theta, \eta)\right|$
- Validation
  - Incomplete pass-through of value added to wages (Kline et al., 2019)

$$\Delta \log w_{ijt} = \Delta \log \mu_{ijt} + \Delta \log vapw_{ijt}$$

  - Responses of firms to competitors' wage changes (Staiger et al., 2010)

$$\Delta \log w_{ij} = \Omega(s_{ij}) \Delta \log vapw_{ij} + (1 - \Omega(s_{ij})) \sum_{k \neq i} \left(\frac{s_{kj}}{1 - s_{ij}}\right) \Delta \log w_{kj}$$

where $\Omega(s_{ij}) = \frac{s_{ij}(\eta-\theta)+\theta(\eta+1)}{[1+(1+\eta)(1-s_{ij})]s_{ij}(\eta-\theta)+\theta(\eta+1)}$

  - Employment and wage effects of mergers (Arnold, 2021)

Jarosch, Nimczik and Sorkin (2021)

- Wage bargaining, not monopsonistic wage setting
- Relate firms' bargaining leverage to firm "size" f_i
- In a standard model, worker's outside option in bargaining is

$$U = b + \beta\left(\lambda \sum_i f_i W_i + (1-\lambda) U\right)$$

- Here firm can remove itself from worker's outside option (outside options are truly outside)

$$U_i = b + \beta\left(\lambda \sum_{j \neq i} f_j W_j + \underline{\lambda} f_i W_i + \left(1 - \lambda\left(1 - f_i\right) - \underline{\lambda} f_i\right) U_i\right)$$

  where $\underline{\lambda}$ is the probability that the worker is the only applicant

- Bargaining happens only within a single worker-firm pair

Conclusion

- Recent resurgent interests in labor market power
  - Many job market papers on this topic as a sign ▸ List
  - Speculations on candidate explanation for falling labor share and rising inequality
  - Worries often heard about mega-firms, demise of unions, anti-competitive labor contracts
- Monopsonistic wage setting or bargaining?
  - Monopsonistic view: markdown (deviation of w from MRPL) measures labor market power
  - Is $w =$ MRPL the most relevant benchmark? If search friction is technological, it does not correspond to the (constrained) efficient benchmark in DMP (Hosios condition).
  - Both are empirically challenging
    - Monopsonistic view: MRPL, hence markdown, is intrinsically unobserved
    - Bargaining view: bargaining power and outside option are difficult to measure
- IO has developed many tools to study market power and strategic interactions during past decades. It might be fruitful for labor and IO economists to talk to each other.

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Appendix

Job Market Papers on Labor Market Power in 2022

- Iain Bamford (Columbia), "Monopsony Power, Spatial Equilibrium, and Minimum Wages"
- Nikhil Datta (UCL), "Local Monopsony Power"
- Mayara Felix (MIT), "Trade, Labor Market Concentration, and Wages"
- Negin Mousavi (Chicago), "Optimal Labor Income Tax, Incomplete Markets, Labor Market Power"
- Preston Mui (Berkeley), "Labor Market Monopsony in the New Keynesian Model: Theory and Evidence"
- Bryan Seegmiller (MIT Sloan), "Valuing Labor Market Power: The Role of Productivity Advantages"
- Benjamin Scuderi (Berkeley), "Bidding for Talent: Equilibrium Wage Dispersion on a High-Wage Online Job Board"
- James Traina (Chicago Booth), "Labor Market Power and Technological Change in US Manufacturing"
- Justin C. Wiltshire (UC Davis), "Walmart Supercenters and Monopsony Power: How a Large, Low-Wage Employer Impacts Local Labor Markets"
- ...

Derivation for Household Problem I

- The intra-temporal Euler equation (i.e., FOCs for consumption and labor supply to firm-ij)

$$w_{ijt} = \frac{\partial \mathbf{n}_{jt}}{\partial n_{ijt}} \frac{\partial \mathbf{N}_t}{\partial \mathbf{n}_{jt}} \left(- \frac{U_N\left(\mathbf{C}_t, \mathbf{N}_t\right)}{U_C\left(\mathbf{C}_t, \mathbf{N}_t\right)} \right)$$

- The labor supply disutility functional form delivers

$$\frac{\partial \mathbf{N}_t}{\partial \mathbf{n}_{jt}} \frac{\mathbf{n}_{jt}}{\mathbf{N}_t} = \left(\frac{\mathbf{n}_{jt}}{\mathbf{N}_t} \right)^{\frac{\theta+1}{\theta}}, \quad \frac{\partial \mathbf{n}_{jt}}{\partial n_{ijt}} \frac{n_{ijt}}{\mathbf{n}_{jt}} = \left(\frac{n_{ijt}}{\mathbf{n}_{jt}} \right)^{\frac{\eta+1}{\eta}}$$

- Therefore

$$\int_0^1 \frac{\partial \mathbf{N}_t}{\partial \mathbf{n}_{jt}} \frac{\mathbf{n}_{jt}}{\mathbf{N}_t} dj = \mathbf{N}_t^{-\frac{\theta+1}{\theta}} \int_0^1 \mathbf{n}_{jt}^{\frac{\theta+1}{\theta}} dj = 1, \quad \sum_{i \in j} \frac{\partial \mathbf{n}_{jt}}{\partial n_{ijt}} \frac{n_{ijt}}{\mathbf{n}_{jt}} = \mathbf{n}_{jt}^{-\frac{\eta+1}{\eta}} \sum_{i \in j} n_{ijt}^{\frac{\eta+1}{\eta}} = 1.$$

Derivation for Household Problem II

- Using these equations we have

$$\mathbf{W}_t\mathbf{N}_t = \int_0^1 \underbrace{\sum_{i\in j}\left(\frac{\partial \mathbf{n}_{jt}}{\partial n_{ijt}}\frac{n_{ijt}}{\mathbf{n}_{jt}}\right)\left(\frac{\partial \mathbf{N}_t}{\partial \mathbf{n}_{jt}}\frac{\mathbf{n}_{jt}}{\mathbf{N}_t}\right)\left(-\frac{U_N(\mathbf{C}_t,\mathbf{N}_t)}{U_C(\mathbf{C}_t,\mathbf{N}_t)}\right)\mathbf{N}_t}_{w_{ijt}n_{ijt}}\, dj = \left(-\frac{U_N(\mathbf{C}_t,\mathbf{N}_t)}{U_C(\mathbf{C}_t,\mathbf{N}_t)}\right)\mathbf{N}_t$$

- Therefore, we have the aggregate labor supply curve $\mathbf{W}_t = -\frac{U_N(\mathbf{C}_t,\mathbf{N}_t)}{U_C(\mathbf{C}_t,\mathbf{N}_t)}$
- The market labor supply curve can be obtained by

$$\mathbf{w}_{jt}\mathbf{n}_{jt} = \sum_{i\in j} w_{ijt}n_{ijt} = \left(\frac{\mathbf{n}_{jt}}{\mathbf{N}_t}\right)^{\frac{\theta+1}{\theta}}\mathbf{W}_t\mathbf{N}_t \Rightarrow \mathbf{n}_{jt} = \left(\frac{\mathbf{w}_{jt}}{\mathbf{W}_t}\right)^{\theta}\mathbf{N}_t$$

- Lastly, the firm-specific labor supply curve

$$w_{ijt}n_{ijt} = \left(\frac{n_{ijt}}{\mathbf{n}_{jt}}\right)^{\frac{\eta+1}{\eta}}\left(\frac{\mathbf{w}_{jt}\mathbf{n}_{jt}}{\mathbf{W}_t\mathbf{N}_t}\right)\mathbf{W}_t\mathbf{N}_t \Rightarrow n_{ijt} = \left(\frac{w_{ijt}}{\mathbf{w}_{jt}}\right)^{\eta}\mathbf{n}_{jt}$$

Microfoundation: Logit and CES

- Worker l's disutility of working h_{lij} hours at firm ij are:

$$v_{lij} = e^{-\xi_{lij}} h_{lij}, \quad \log v_{lij} = \log h_{ij} - \xi_{lij}$$

- The random utility term ξ_{lij} is distributed iid from a multi-variate Gumbel distribution

$$F\left(\xi_{i1}, \ldots, \xi_{NJ}\right) = \exp\left[-\sum_{ij} e^{-(1+\eta)\xi_{ij}}\right]$$

- Suppose each worker must earn $y_l \sim G(y)$ such that h_{lij} satisfies $y_l = w_{ij} h_{lij}$. Worker l solves

$$\min_{ij}\left\{\log h_{ij} - \xi_{lij}\right\} \equiv \max_{ij}\left\{\log w_{ij} - \log y_l + \xi_{lij}\right\}$$

- This problem delivers the following choice probability for worker l

$$\Pr\left(w_{ij}, w_{-ij}; l\right) = \frac{w_{ij}^{1+\eta}}{\sum_{ij} w_{ij}^{1+\eta}}$$

Microfoundation: Logit and CES

- Total labor supply to firm ij is

$$n_{ij} = \int \Pr\left(w_{ij}, w_{-ij}; l\right) \times h_{lij} dG\left(y_l\right) = \frac{w_{ij}^{\eta}}{\sum_{ij} w_{ij}^{1+\eta}} \underbrace{\int y_l G\left(y_l\right)}_{Y}$$

- Aggregating this expression we obtain the obvious result that $\sum_{ij} w_{ij} n_{ij} = Y$

- Define indexes $\mathbf{W} := \left[\sum_{ij} w_{ij}^{1+\eta}\right]^{\frac{1}{1+\eta}}$, $\mathbf{N} := \left[\sum_{ij} n_{ij}^{\frac{\eta+1}{\eta}}\right]^{\frac{\eta}{\eta+1}}$. It holds that $\mathbf{WN} = Y$.

- Thus

$$n_{ij} = \frac{w_{ij}^{\eta}}{\sum_{ij} w_{ij}^{1+\eta}} Y = \frac{w_{ij}^{\eta}}{\mathbf{W}^{1+\eta}} \mathbf{WN} = \left(\frac{w_{ij}}{\mathbf{W}}\right)^{\eta} \mathbf{N}$$

Microfoundation: Nested Logit and Nested CES

- Preference shock distribution $F\left(\xi_{i1},\ldots,\xi_{NJ}\right)=\exp\left[-\sum_{j=1}^{J}\left(\sum_{i=1}^{M_j}e^{-(1+\eta)\xi_{ij}}\right)^{\frac{1+\theta}{1+\eta}}\right]$
- The choice probabilities can be expressed as

$$\Pr\left(w_{ij},w_{-ij}\right)=\underbrace{\frac{w_{ij}^{1+\eta}}{\sum_{i=1}^{M_j}w_{ij}^{1+\eta}}}_{\Pr(\text{Choose firm } i|\text{Choose market } j)}\times\underbrace{\frac{\left[\sum_{i=1}^{M_j}w_{ij}^{1+\eta}\right]^{\frac{1+\theta}{1+\eta}}}{\sum_{l=1}^{J}\left[\sum_{k=1}^{M_l}w_{kl}^{1+\eta}\right]^{\frac{1+\theta}{1+\eta}}}}_{\Pr(\text{Choose market } j)}$$

- Following the same steps as before

$$n_{ij}=\frac{w_{ij}^{\eta}}{\sum_{i=1}^{M_j}w_{ij}^{1+\eta}}\frac{\left[\sum_{i=1}^{M_j}w_{ij}^{1+\eta}\right]^{\frac{1+\theta}{1+\eta}}}{\sum_{l=1}^{J}\left[\sum_{k=1}^{M_l}w_{kl}^{1+\eta}\right]^{\frac{1+\theta}{1+\eta}}}Y$$

- See Dupuy and Galichon (2014) for the formalism of the continuum limit